FINAL JEE–MAIN EXAMINATION – SEPTEMBER, 2020
(Held On Friday 04th SEPTEMBER, 2020)   TIME : 3 PM to 6 PM

1. The function $f(x) = \begin{cases} \dfrac{\pi}{4} + \tan^{-1} x, |x| \le 1 \\ \dfrac{1}{2}(|x|-1), |x| > 1 \end{cases}$ is :

(1) continuous on R–{1} and differentiable on R – {–1, 1}.

(2) both continuous and differentiable on R – {–1}.

(3) continuous on R – {–1} and differentiable on R – {–1, 1}.

(4) both continuous and differentiable on R –{1}

Official Ans. by NTA (1)

2. Let $\overset{50}{\underset{i=1}{U}} X_i = \overset{n}{\underset{i=1}{U}} Y_i = T$, where each X_i contains 10 elements and each Y_i contains 5 elements. If each element of the set T is an element of exactly 20 of sets X_i's and exactly 6 of sets Y_i's, then n is equal to :

(1) 45   (2) 15

(3) 50   (4) 30

Official Ans. by NTA (4)

3. Let $\lambda \ne 0$ be in R. If α and β are the roots of the equation, $x^2 - x + 2\lambda = 0$ and α and γ are the roots of the equation, $3x^2 - 10x + 27\lambda = 0$, then $\dfrac{\beta\gamma}{\lambda}$ is equal to :

(1) 36   (2) 27

(3) 9   (4) 18

Official Ans. by NTA (4)

4. The solution of the differential equation

$\dfrac{dy}{dx} - \dfrac{y+3x}{\log_e(y+3x)} + 3 = 0$ is :-

(where C is a constant of integration.)

(1) x–2 $\log_e$(y+3x)=C

(2) x–$\log_e$(y+3x)=C

(3) x–$\dfrac{1}{2}$ $(\log_e(y+3x))^2$ = C

(4) y + 3x–$\dfrac{1}{2}$ $(\log_e x)^2$ = C

Official Ans. by NTA (3)

5. Let a_1, a_2..., a_n be a given A.P. whose common difference is an integer and $S_n = a_1 + a_2 + .. + a_n$. If $a_1 = 1$, $a_n = 300$ and $15 \le n \le 50$, then the ordered pair (S_{n-4}, a_{n-4}) is equal to :

(1) (2480, 249)   (2) (2490, 249)

(3) (2490, 248)   (4) (2480, 248)

Official Ans. by NTA (3)

6. The distance of the point $(1, -2, 3)$ from the plane x–y+z = 5 measured parallel to the line $\dfrac{x}{2} = \dfrac{y}{3} = \dfrac{z}{-6}$ is :

(1) 7   (2) 1   (3) $\dfrac{1}{7}$ (4) $\dfrac{7}{5}$

Official Ans. by NTA (2)

7. Let $f : (0, \infty) \to (0, \infty)$ be a differentiable function such that f(1) = e and

$\lim\limits_{t \to x} \dfrac{t^2 f^2(x) - x^2 f^2(t)}{t - x} = 0$

If f(x) = 1, then x is equal to :

(1) 2e   (2) $\dfrac{1}{2e}$ (3) e   (4) $\dfrac{1}{e}$

Official Ans. by NTA (4)

8. If the system of equations

x + y + z = 2

2x + 4y − z = 6

3x + 2y + λz = μ

has infinitely many solutions, then :

(1) λ − 2μ = −5   (2) 2λ − μ = 5

(3) 2λ + μ = 14   (4) λ + 2μ = 14

Official Ans. by NTA (3)

9. The minimum value of $2^{\sin x} + 2^{\cos x}$ is :-

(1) $2^{1-\frac{1}{\sqrt{2}}}$ (2) $2^{-1+\sqrt{2}}$

(3) $2^{1-\sqrt{2}}$ (4) $2^{-1+\frac{1}{\sqrt{2}}}$

Official Ans. by NTA (1)

10. $\int_{\pi/6}^{\pi/3} \tan^3 x \cdot \sin^2 3x (2\sec^2 x \cdot \sin^2 3x + 3\tan x \cdot \sin 6x)\,dx$

is equal to :

(1) $\dfrac{9}{2}$ (2) $-\dfrac{1}{9}$ (3) $-\dfrac{1}{18}$ (4) $\dfrac{7}{18}$

Official Ans. by NTA (3)

11. The circle passing through the intersection of the circles, $x^2 + y^2 - 6x = 0$ and $x^2 + y^2 - 4y = 0$, having its centre on the line, $2x - 3y + 12 = 0$, also passes through the point :

(1) (1, −3)   (2) (−1, 3)

(3) (−3, 1)   (4) (−3, 6)

Official Ans. by NTA (4)

12. The angle of elevation of a cloud C from a point P, 200 m above a still lake is 30º. If the angle of depression of the image of C in the lake from the point P is 60º, then PC (in m) is equal to :

(1) 400   (2) $400\sqrt{3}$

(3) 100   (4) $200\sqrt{3}$

Official Ans. by NTA (1)

13. If a and b are real numbers such that

$(2 + \alpha)^4 = a + b\alpha$, where $\alpha = \dfrac{-1+i\sqrt{3}}{2}$, then

a + b is equal to :

(1) 57   (2) 33   (3) 24   (4) 9

Official Ans. by NTA (4)

14. In a game two players A and B take turns in throwing a pair of fair dice starting with player A and total of scores on the two dice, in each throw is noted. A wins the game if he throws a total of 6 before B throws a total of 7 and B wins the game if he throws a total of 7 before A throws a total of six The game stops as soon as either of the players wins. The probability of A winning the game is :

(1) $\dfrac{31}{61}$ (2) $\dfrac{5}{6}$ (3) $\dfrac{5}{31}$ (4) $\dfrac{30}{61}$

Official Ans. by NTA (4)

15. Let x = 4 be a directrix to an ellipse whose centre is at the origin and its eccentricity is $\dfrac{1}{2}$.

If P (1, β), β > 0 is a point on this ellipse, then the equation of the normal to it at P is :-

(1) 7x − 4y = 1   (2) 4x − 2y = 1

(3) 4x − 3y = 2   (4) 8x − 2y = 5

Official Ans. by NTA (2)

16. Contrapositive of the statement:

'If a function f is differentiable at a, then it is also continuous at a', is :-

(1) If a function f is continuous at a, then it is not differentiable at a.

(2) If a function f is not continuous at a, then it is differentiable at a.

(3) If a function f is not continuous at a, then it is not differentiable at a.

(4) If a function f is continuous at a, then it is differentiable at a.

Official Ans. by NTA (3)

17. The area (in sq. units) of the largest rectangle ABCD whose vertices A and B lie on the x-axis and vertices C and D lie on the parabola, $y = x^2 - 1$ below the x-axis, is :

(1) $\dfrac{4}{3\sqrt{3}}$ (2) $\dfrac{1}{3\sqrt{3}}$ (3) $\dfrac{4}{3}$ (4) $\dfrac{2}{3\sqrt{3}}$

Official Ans. by NTA (1)

18. If for some positive integer n, the coefficients of three consecutive terms in the binomial expansion of $(1+x)^{n+5}$ are in the ratio 5 : 10 : 14, then the largest coefficient in this expansion is :-
(1) 792   (2) 252   (3) 462   (4) 330
Official Ans. by NTA (3)

19. If the perpendicular bisector of the line segment joining the points P (1, 4) and Q (k, 3) has y-intercept equal to –4, then a value of k is :-
(1) $\sqrt{15}$ (2) –2   (3) $\sqrt{14}$ (4) –4
Official Ans. by NTA (4)

20. Suppose the vectors x_1, x_2 and x_3 are the solutions of the system of linear equations, Ax = b when the vector b on the right side is equal to b_1, b_2 and b_3 respectively. If

$$x = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}, x_2 = \begin{bmatrix} 0 \\ 2 \\ 1 \end{bmatrix}, x_3 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}, b_1 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix},$$

$$b_2 = \begin{bmatrix} 0 \\ 2 \\ 0 \end{bmatrix} \text{ and } b_3 = \begin{bmatrix} 0 \\ 0 \\ 2 \end{bmatrix}, \text{ then the determinant of}$$

A is equal to :-

(1) $\dfrac{1}{2}$ (2) 4   (3) $\dfrac{3}{2}$ (4) 2

Official Ans. by NTA (4)

21. A test consists of 6 multiple choice questions, each having 4 alternative answers of which only one is correct. The number of ways, in which a candidate answers all six questions such that exactly four of the answers are correct, is ____
Official Ans. by NTA (135)

22. Let PQ be a diameter of the circle $x^2+y^2=9$. If α and β are the lengths of the perpendiculars from P and Q on the straight line, x + y = 2 respectively, then the maximum value of αβ is ____
Official Ans. by NTA (7)

23. Let {x} and [x] denote the fractional part of x and the greatest integer ≤ x respectively of a real number x. If $\int_0^n \{x\}dx, \int_0^n [x]dx$ and $10(n^2 - n)$, $(n \in N, n > 1)$ are three consecutive terms of a G.P., then n is equal to____
Official Ans. by NTA (21)

24. If $\vec{a} = 2\hat{i} + \hat{j} + 2\hat{k}$, then the value of $\left|\hat{i} \times (\vec{a} \times \hat{i})\right|^2 + \left|\hat{j} \times (\vec{a} \times \hat{j})\right|^2 + \left|\hat{k} \times (\vec{a} \times \hat{k})\right|^2$ is equal to ____
Official Ans. by NTA (18)

25. If the variance of the following frequency distribution :

Class	10–20	20–30	30–40
Frequency	2	x	2

is 50, then x is equal to ____
Official Ans. by NTA (4)