



FINAL JEE–MAIN EXAMINATION – SEPTEMBER, 2020

(On Friday 04th SEPTEMBER, 2020) TIME : 9 AM to 12 PM

MATHEMATICS

1. If $A = \begin{bmatrix} \cos \theta & i \sin \theta \\ i \sin \theta & \cos \theta \end{bmatrix}$, $\left(\theta = \frac{\pi}{24} \right)$ and

$A^5 = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$, where $i = \sqrt{-1}$, then which one

of the following is not true?

(1) $0 \leq a^2 + b^2 \leq 1$ (2) $a^2 - d^2 = 0$

(3) $a^2 - b^2 = \frac{1}{2}$ (4) $a^2 - c^2 = 1$

Official Ans. by NTA (3)

2. Let $[t]$ denote the greatest integer $\leq t$. Then the equation in x , $[x]^2 + 2[x + 2] - 7 = 0$ has :

- (1) no integral solution
(2) exactly four integral solutions
(3) exactly two solutions
(4) infinitely many solutions

Official Ans. by NTA (4)

3. Let α and β be the roots of $x^2 - 3x + p = 0$ and γ and δ be the roots of $x^2 - 6x + q = 0$. If $\alpha, \beta, \gamma, \delta$ form a geometric progression. Then ratio $(2q + p) : (2q - p)$ is :

- (1) 3 : 1 (2) 33 : 31
(3) 9 : 7 (4) 5 : 3

Official Ans. by NTA (3)

4. Let $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ ($a > b$) be a given ellipse, length of whose latus rectum is 10. If its eccentricity is the maximum value of the function, $\phi(t) = \frac{5}{12} + t - t^2$, then $a^2 + b^2$ is equal to :

- (1) 126 (2) 135
(3) 145 (4) 116

Official Ans. by NTA (1)

TEST PAPER WITH ANSWER

5. A triangle ABC lying in the first quadrant has two vertices as A(1, 2) and B(3, 1). If $\angle BAC = 90^\circ$, and $\text{ar}(\triangle ABC) = 5\sqrt{5}$ sq. units, then the abscissa of the vertex C is :

- (1) $2 + \sqrt{5}$ (2) $1 + \sqrt{5}$
(3) $1 + 2\sqrt{5}$ (4) $2\sqrt{5} - 1$

Official Ans. by NTA (3)

6. Let $f(x) = |x - 2|$ and $g(x) = f(f(x))$, $x \in [0, 4]$.

Then $\int_0^3 (g(x) - f(x)) dx$ is equal to :

- (1) $\frac{3}{2}$ (2) 0
(3) $\frac{1}{2}$ (4) 1

Official Ans. by NTA (4)

7. Given the following two statements :

(S₁) : $(q \vee p) \rightarrow (p \leftrightarrow \sim q)$ is a tautology.

(S₂) : $\sim q \wedge (\sim p \leftrightarrow q)$ is a fallacy.

Then :

- (1) only (S₁) is correct.
(2) both (S₁) and (S₂) are correct.
(3) both (S₁) and (S₂) are not correct.
(4) only (S₂) is correct.

Official Ans. by NTA (3)

8. Let $P(3, 3)$ be a point on the hyperbola,

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1. \text{ If the normal to it at } P \text{ intersects}$$

the x -axis at $(9, 0)$ and e is its eccentricity, then the ordered pair (a^2, e^2) is equal to :

(1) $\left(\frac{9}{2}, 3\right)$ (2) $\left(\frac{9}{2}, 2\right)$

(3) $\left(\frac{3}{2}, 2\right)$ (4) $(9, 3)$

Official Ans. by NTA (1)

9. Let $f(x) = \int \frac{\sqrt{x}}{(1+x)^2} dx$ ($x \geq 0$). Then $f(3) - f(1)$ is equal to :

(1) $-\frac{\pi}{6} + \frac{1}{2} + \frac{\sqrt{3}}{4}$ (2) $\frac{\pi}{6} + \frac{1}{2} - \frac{\sqrt{3}}{4}$

(3) $-\frac{\pi}{12} + \frac{1}{2} + \frac{\sqrt{3}}{4}$ (4) $\frac{\pi}{12} + \frac{1}{2} - \frac{\sqrt{3}}{4}$

Official Ans. by NTA (4)

10. A survey shows that 63% of the people in a city read newspaper A whereas 76% read newspaper B. If $x\%$ of the people read both the newspapers, then a possible value of x can be:

(1) 65 (2) 37

(3) 29 (4) 55

Official Ans. by NTA (4)

11. Let $u = \frac{2z+i}{z-ki}$, $z = x + iy$ and $k > 0$. If the curve represented by $\text{Re}(u) + \text{Im}(u) = 1$ intersects the y -axis at the points P and Q where $PQ = 5$, then the value of k is :

(1) $3/2$ (2) 4

(3) 2 (4) $1/2$

Official Ans. by NTA (3)

12. Let x_0 be the point of local maxima of $f(x) = \vec{a} \cdot (\vec{b} \times \vec{c})$, where $\vec{a} = x\hat{i} - 2\hat{j} + 3\hat{k}$, $\vec{b} = -2\hat{i} + x\hat{j} - \hat{k}$ and $\vec{c} = 7\hat{i} - 2\hat{j} + x\hat{k}$. Then the value of $\vec{a} \cdot \vec{b} + \vec{b} \cdot \vec{c} + \vec{c} \cdot \vec{a}$ at $x = x_0$ is :

(1) -30 (2) 14
(3) -4 (4) -22

Official Ans. by NTA (4)

13. Two vertical poles $AB = 15$ m and $CD = 10$ m are standing apart on a horizontal ground with points A and C on the ground. If P is the point of intersection of BC and AD , then the height of P (in m) above the line AC is :

(1) $20/3$ (2) 5
(3) $10/3$ (4) 6

Official Ans. by NTA (4)

14. The mean and variance of 8 observations are 10 and 13.5, respectively. If 6 of these observations are 5, 7, 10, 12, 14, 15, then the absolute difference of the remaining two observations is :

(1) 7 (2) 3
(3) 5 (4) 9

Official Ans. by NTA (1)

15. The integral $\int \left(\frac{x}{x \sin x + \cos x} \right)^2 dx$ is equal to :

(where C is a constant of integration)

(1) $\sec x + \frac{x \tan x}{x \sin x + \cos x} + C$

(2) $\sec x - \frac{x \tan x}{x \sin x + \cos x} + C$

(3) $\tan x + \frac{x \sec x}{x \sin x + \cos x} + C$

(4) $\tan x - \frac{x \sec x}{x \sin x + \cos x} + C$

Official Ans. by NTA (4)

16. If

$1 + (1 - 2^2 \cdot 1) + (1 - 4^2 \cdot 3) + (1 - 6^2 \cdot 5) + \dots + (1 - 20^2 \cdot 19)$
 $= \alpha - 220\beta$, then an ordered pair (α, β) is equal to :

- (1) (10, 97) (2) (11, 103)
 (3) (10, 103) (4) (11, 97)

Official Ans. by NTA (2)

17. Let $y = y(x)$ be the solution of the differential equation, $xy' - y = x^2(x \cos x + \sin x)$, $x > 0$.

If $y(\pi) = \pi$, then $y''\left(\frac{\pi}{2}\right) + y\left(\frac{\pi}{2}\right)$ is equal to :

- (1) $2 + \frac{\pi}{2}$ (2) $1 + \frac{\pi}{2}$
 (3) $1 + \frac{\pi}{2} + \frac{\pi^2}{4}$ (4) $2 + \frac{\pi}{2} + \frac{\pi^2}{4}$

Official Ans. by NTA (1)

18. The value of $\sum_{r=0}^{20} {}^{50-r}C_6$ is equal to :

- (1) ${}^{51}C_7 + {}^{30}C_7$ (2) ${}^{51}C_7 - {}^{30}C_7$
 (3) ${}^{50}C_7 - {}^{30}C_7$ (4) ${}^{50}C_6 - {}^{30}C_6$

Official Ans. by NTA (2)

19. Let f be a twice differentiable function on $(1, 6)$. If $f(2) = 8$, $f'(2) = 5$, $f'(x) \geq 1$ and $f''(x) \geq 4$, for all $x \in (1, 6)$, then :

- (1) $f(5) \leq 10$ (2) $f'(5) + f''(5) \leq 20$
 (3) $f(5) + f'(5) \geq 28$ (4) $f(5) + f''(5) \leq 26$

Official Ans. by NTA (3)

20. If $(a + \sqrt{2} b \cos x)(a - \sqrt{2} b \cos y) = a^2 - b^2$,

where $a > b > 0$, then $\frac{dx}{dy}$ at $\left(\frac{\pi}{4}, \frac{\pi}{4}\right)$ is :

- (1) $\frac{a-b}{a+b}$ (2) $\frac{a+b}{a-b}$
 (3) $\frac{2a+b}{2a-b}$ (4) $\frac{a-2b}{a+2b}$

Official Ans. by NTA (2)

21. If the system of equations

$$x - 2y + 3z = 9$$

$$2x + y + z = b$$

$$x - 7y + az = 24,$$

has infinitely many solutions, then $a - b$ is equal to _____.

Official Ans. by NTA (5)

22. The probability of a man hitting a target is $\frac{1}{10}$.

The least number of shots required, so that the probability of his hitting the target at least once

is greater than $\frac{1}{4}$, is _____.

Official Ans. by NTA (3)

23. Suppose a differentiable function $f(x)$ satisfies the identity $f(x + y) = f(x) + f(y) + xy^2 + x^2y$,

for all real x and y . If $\lim_{x \rightarrow 0} \frac{f(x)}{x} = 1$, then $f'(3)$

is equal to _____.

Official Ans. by NTA (10)

24. Let $(2x^2 + 3x + 4)^{10} = \sum_{r=0}^{20} a_r x^r$. Then $\frac{a_7}{a_{13}}$ is

equal to _____.

Official Ans. by NTA (8)

25. If the equation of a plane P , passing through the intersection of the planes, $x + 4y - z + 7 = 0$ and $3x + y + 5z = 8$ is $ax + by + 6z = 15$ for some $a, b \in \mathbb{R}$, then the distance of the point $(3, 2, -1)$ from the plane P is _____.

Official Ans. by NTA (3)