Mathematics JEE Solutions 2022

Mathematics

1. Find the remainder when 3^{2022} is divided by 5.

(1) 1 (2) 2

(3) 4 (4) 0

Sol. Answer (3)

$3^{2022} = (9)^{1011}$

$= (10 - 1)^{1011}$

$= -(1 - 10)^{1011}$

$= \left[{}^{1011}C_0 - {}^{1011}C_1 \cdot 10 + {}^{1011}C_2 10^2 + \cdots\right]$

$= -[1 + 10k]$

$= -[1 + 5k_2]$

$\therefore$ 3^{2022} is divided by 5, –1 is the negative.

Remainder so positive remainder is 4.

2. If sum of square of reciprocal of roots 'α' and 'β' of equation $3x^2 - \lambda x + 1 = 0$ is 15, then find $6(\alpha^3 + \beta^3)^2$.

(1) $\frac{202}{3}$ (2) $\frac{200}{9}$

(3) $\frac{224}{9}$ (4) $\frac{221}{3}$

Sol. Answer (3)

$$3x^2 - \lambda x + 1 = 0 \ \langle^{\alpha}_{\beta}$$

$$\alpha + \beta = \frac{\lambda}{3}$$

$$\alpha\beta = \frac{1}{3}$$

$$(\alpha+\beta)^2 - 2\alpha\beta = 15(\alpha\beta)^2$$

$$\frac{1}{\alpha^2} + \frac{1}{\beta^2} = 15$$

$$\Rightarrow \quad \frac{\alpha^2 + \beta^2}{(\alpha\beta)^2} = 15$$

$$\Rightarrow \quad \frac{\lambda^2}{9} - \frac{2}{3} = \frac{15}{9}$$

$$\Rightarrow \quad \frac{\lambda^2 - 6}{9} = \frac{15}{9}$$

$$\lambda^2 = 21$$

Now, $6(\alpha^3 + \beta)^2 = 6((\alpha+\beta)(\alpha^2 + \beta^2 - \alpha\beta))^2$

$$= 6\left(\left(\frac{\lambda}{3}\right)\left((\alpha+\beta)^2 - 3\alpha\beta\right)\right)^2$$

$$= 6\left(\left(\frac{\lambda}{3}\right)\left(\frac{\lambda^2}{9} - 1\right)\right)^2$$

$$= 6\frac{\lambda^2}{9} \cdot \left(\frac{21}{9} - 1\right)^2 = 26 \cdot \frac{21}{9} \cdot \frac{12}{9^2}$$

$$= 14 \cdot \frac{12}{9} \cdot \frac{12}{9}$$

$$= \frac{14 \times 16}{9} = \frac{224}{9}$$

3. If $(\tan^{-1} x)^3 + (\cot^{-1} x)^3 = k\pi^3$, then find the range of k.

(1) $\left(\frac{1}{32}, \frac{9}{8}\right)$ (2) $\left[\frac{1}{32}, \frac{7}{8}\right)$

(3) $\left[\frac{1}{32}, \frac{9}{8}\right]$ (4) $\left[\frac{1}{32}, 1\right]$

Sol. Answer (2)

$(\tan^{-1} x)^3 + (\cot^{-1} x)^3 = (\tan^{-1} x + \cot^{-1} x)^3$
$-3\tan^{-1} x \cdot \cot^{-1} x(\tan^{-1} x + \cot^{-1} x)$

$$= \left(\frac{\pi}{2}\right)^3 - 3\tan^{-1} x \cdot \cot^{-1} x \cdot \left(\frac{\pi}{2}\right)$$

$$= \left(\frac{\pi}{2}\right)\left[\frac{\pi^2}{4} - 3\tan^{-1} x\left(\frac{\pi}{2} - \tan^{-1} x\right)\right]$$

$$= \frac{\pi}{2}\left[\frac{\pi^2}{4} + 3(\tan^{-1}x)^2 - \frac{3\pi}{2}\tan^{-1}x\right]$$

$$= \frac{\pi}{2}\left[3\left(\tan^{-1}x - \frac{\pi}{4}\right)^2 + \frac{\pi^2}{16}\right]$$

$\because \tan^{-1}x \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$, so $\left(\tan^{-1}x - \frac{\pi}{4}\right)^2 \in \left[0, \frac{9\pi^2}{16}\right)$

Hence $(\tan^{-1}x)^3 + (\cot^{-1}x)^3 \in \left[\frac{\pi^3}{32}, \frac{7\pi^3}{8}\right)$

$\Rightarrow \quad k \in \left[\frac{1}{32}, \frac{7}{8}\right)$

4. If $f(\theta) = \sin\theta + \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} (\sin\theta + t\cos\theta)f(t)\,d\theta,$ then $\left|\int_0^{\pi/2} f(\theta)d\theta\right|$

(1) $1 + \pi t\, f(t)$ (2) $1 - \pi t\, f(t)$

(3) $1 + \pi^2 t\, f(t)$ (4) $-1 + \pi t \cdot f(t)$

Sol. Answer (1)

$$f(\theta) = \sin\theta + \int_{-\pi/2}^{\pi/2} (\sin\theta + t\cos\theta)(f(t)d\theta$$

$$f(\theta) = \sin\theta + \int_{-\pi/2}^{\pi/2} \sin\theta d\theta + tf(t)\int_{-\pi/2}^{\pi/2} \cos\theta d\theta$$

$$f(\theta) = \sin\theta + 0 + t\,f(t)\cdot 2$$

$$f(\theta) = \sin\theta + 2t\,f(t)$$

Now $\left|\int_0^{\pi/2} f(\theta)d\theta\right| = \left|\int_0^{\pi/2} \sin\theta d\theta + 2t\,f(t)\int_0^{\pi/2} 1d\theta\right|$

$$= |1 + \pi t\,f(t)|$$

5. $\langle a_i \rangle$ sequence is an A.P. with common difference is 1 and $\sum_{i=1}^{n} a_i = 192, \sum_{i=1}^{n/2} a_{2i} = 120$ then, find the value of n, where n is an even integer.

(1) 48 (2) 96

(3) 18 (4) 36

Sol. Answer (2)

Let $n = 2m$ $m \in N$ and a be the first term

$\sum_{i=1}^{2m} = 192$ …(1)

$d = 1$

$\Rightarrow \frac{2m}{2}[2a + (2m-1)1] = 192$

$\Rightarrow m(2a + 2m - 1) = 192$ …(3)

$\sum_{i=1}^{m} a_{2i} = 120$ …(2)

$\frac{2m}{2}[2a + (m-1)2] = 120$

$m(a_2 + m - 1) = 120$

$m(a + m) = 120$ …(4)

$(3/4) \Rightarrow \frac{2a + 2m - 1}{a + m} = \frac{192}{120} = \frac{8}{5}$

$\Rightarrow 100 + 10m - 5 = 8a + 8m$

$\Rightarrow 2a + 2m = 5$ put in (3)

$m(5 - 1) = 192$

$\Rightarrow m = \frac{192}{4} = 48$

So $n = 2m = 2 \times 48 = 96$

6. If $A = \begin{bmatrix} 1 & 0 & a \\ 1 & 1 & 0 \\ -1 & 0 & 1 \end{bmatrix}$, where '$a$' is odd value from 1 to 50 and $\sum_{a=1}^{50} |\text{adj } A| = 100k$, then value of k is

(1) $\frac{1723}{2}$ (2) $\frac{1717}{2}$

(3) $\frac{1719}{4}$ (4) $\frac{1821}{4}$

Sol. Answer (4)

$|A| = (a+1)$

$\sum_{a=1}^{50} |\text{Adj } A| = \sum_{a=1}^{50} (a+1)^2 = 100\,k$

$\Rightarrow 1 + 2^2 + \cdots + 51^2 = 100k + 1$

$\Rightarrow \frac{51 \times 52 \times 103}{6} - 1 = 100k$

$\Rightarrow k = \frac{45525}{100} = \frac{1821}{4}$

7. A tangent $ax - \mu y = 2$ to hyperbola $\frac{a^2x^2}{\lambda^2} - \frac{b^2y^2}{1} = 4$, then the value of $\left(\frac{\lambda}{a}\right)^2 - \left(\frac{\mu}{b}\right)^2$ is

(1) 0 (2) 1

(3) 2 (4) 3

Sol. Answer (2)

Condition for tangent is $a^2l^2 - b^2m^2 = n^2$

$$\frac{4\lambda^2}{a^4} \times a^2 - \frac{4}{b^2} \times (-\mu)^2 = 2^2$$

$$\Rightarrow \frac{\lambda^2}{a^2} - \frac{\mu^2}{b^2} = 1$$

8. A tangent at (x, y) to the cure $y = x^3 + 2x^2 + 4$ and passes through origin then (x_1, y_1) is

(1) (0, 4) (2) (–1, 5)

(3) (1, 7) (4) (2, 20)

Sol. Answer (3)

$y = x^3 + 2x^2 + 4$

when tangent is drawn from origin,

$xf'(x) = f(x)$

$x(3x^2 + 4x) = x^3 + 2x^2 + 4$

$2x^3 + 2x^2 - 4 = 0$

$x^3 + x^2 - 2 = 0$

$(x-1)(x^2 + \underset{\substack{\downarrow \\ \text{no solutions}}}{2x} + 2) = 0$

$x = 1 \Rightarrow y = 7$

$P = (1, 7)$

9. Find the domain of $\dfrac{\cos^{-1}\left(\frac{x^2-5x+6}{x^2-9}\right)}{\ln(x^2)}$

(1) $x \in \left(-\infty, \frac{5}{2}\right]$

(2) $x \in \left(-\infty, \frac{5}{2}\right] - \{-3, 0\}$

(3) $x \in \left(-\infty, \frac{5}{2}\right] - \{0\}$

(4) $x \in \left(-\infty, \frac{5}{2}\right] - \{-3\}$

Sol. Answer (*)

$$f(x) = \frac{\cos^{-1}\left(\frac{x^2-5x+6}{x^2-9}\right)}{\ln x^2}$$

$x^2 \neq 1 \Rightarrow x \neq \pm 1$ and $x \neq 0$

Now, $-1 \le \frac{x^2-5x+6}{x^2-9} \le 1$

Case 1:

$$-1 \le \frac{x^2-5x+6}{x^2-9} \le 1$$

$$\frac{x^2-5x+6}{x^2-9} + 1 \ge 0$$

$$\Rightarrow \frac{x^2-5x+6+x^2-9}{x^2-9} \ge 0$$

$$\Rightarrow \frac{2x^2-5x-3}{(x-3)(x+3)} \ge 0$$

$$\Rightarrow \frac{2x^2-6x+x-3}{(x-3)(x+3)} \ge 0$$

$$\Rightarrow \frac{2x(x-3)+(x-3)}{(x-3)(x+3)} \ge 0$$

$$\Rightarrow \frac{(x-3)+(2x+1)}{(x-3)(x+3)} \ge 0$$

$x \neq 3, \quad \frac{2x+1}{x+3} \ge 0$

+ , – , +

–3 $-\frac{1}{2}$

$x \in (-\infty, -3) \cup \left[-\frac{1}{2}, \infty\right) - \{3\}$

Case 2:

$$\frac{x^2-5x+6}{x^2-9} \le 1$$

$$\Rightarrow \frac{x^2-5x+6-x^2+9}{x^2-9} \le 0$$

$$\Rightarrow \frac{15-5x}{(x-3)(x+3)} \le 0$$

$$\Rightarrow \frac{5(3-x)}{(x-3)(x+3)} \le 0$$

$$\Rightarrow \frac{(x-3)}{(x-3)(x+3)} \ge 0$$

$x \neq 3 \quad x+3 \ge 0$

$x \ge -3$

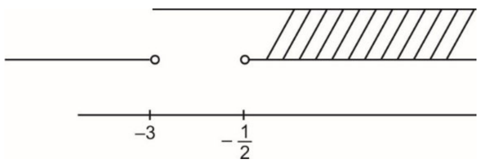
$x \in [-3, \infty) - \{3\}$

(1) $\cap$ (2)

Case-1 $(-\infty, -3) \cup \left[-\frac{1}{2}, \infty\right) - \{3\}$

Case-2 $x \in [-3, \infty) - \{3\}$

t $x = \pm 1$ and $x \pm 0$



$x \in \left[-\frac{1}{2}, \infty\right) - \{0, 1, 3\}$

10. Solution of differential equation $x\frac{dy}{dx} - 2y$ is

(1) $xy = c$ (2) $y = cx^2$

(3) $cx = y^2$ (4) $x^2 = cy^2$

Sol. Answer (2)

$$\frac{x\,dy}{dx} = 2y$$

$$\Rightarrow \int \frac{dy}{y} = \int \frac{2dx}{x}$$

$$\ln y = 2\ln(xc)$$

$$y = c^2x^2$$

$$\Rightarrow \boxed{y = cx^2}$$

11. Consider a set $\Delta \in \{\vee, \wedge, \Rightarrow, \Leftrightarrow\}$ and $P \,\Delta\, q \Rightarrow (\sim P \,\Delta\, q) \,\Delta\, (\sim q \,\Delta\, P)$ is a tautology. Then number of arrangement is

(1) 1 (2) 2

(3) 3 (4) 4

Sol. Answer (3)

If $P \,\Delta\, q \Rightarrow (\sim P \,\Delta\, q) \,\Delta\, (\sim q) \,\Delta\, P)$ is tautology

So, $P \,\Delta\, q = T$ and $(\sim P \,\Delta\, q) \,\Delta\, (\sim q \,\Delta\, P)$ is T

$P \,\Delta\, q = F$ and $(\sim P \,\Delta\, q) \,\Delta\, (\sim q \,\Delta\, P)$ is T

$P \,\Delta\, q = F$ and $(\sim P \,\Delta\, q) \,\Delta\, (\sim q \,\Delta\, P)$ is F

(1) for $D \equiv \vee$, if is a tautology

(2) for $D \equiv \wedge$, $P \wedge q = T \Rightarrow P = q = T$, not a tautology

$P \wedge q = f \Rightarrow P = f$ and $q = T$, not a tautology

$P = T$ and $q = F$

(3) for $D \equiv \Rightarrow$, $P \to q = T$ for $P = T$, $q = T$, is a tautology

$P = F_1$, $q = T$, is a tautology

$P = F_1$, $q = F$, is a tautology

(4) $D \equiv$, $P \to q\ T$

$P = T_1\ \theta = T$	$P = F$ and $Q = F$
RHS $= (F \Leftrightarrow T) \Leftrightarrow (F \Leftrightarrow T)$	RHS $= (T \Leftrightarrow F) \Leftrightarrow (T \Leftrightarrow F)$
$= T$	$= F \Leftrightarrow F$
= Tautology	= Tautology

12. The Boolen expression: $(p \Rightarrow q) \wedge (q \Rightarrow \sim p)$ is equivalent to

(1) $\sim q$ (2) q

(3) $\sim p$ (4) p

Sol. Answer (3)

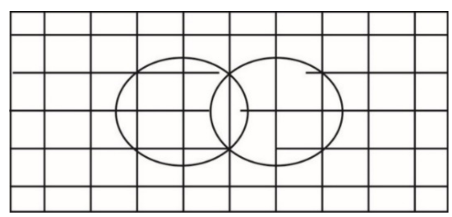
$(P \Rightarrow q) \wedge q \Rightarrow \sim P)$

$= (\sim P \vee q) \wedge (\sim q \vee \sim P)$

$= (\sim P \vee q) \wedge \sim(P \wedge q)$

$= \sim$P

(from venn diagram)



13. Find number of solution in $\left[0, \frac{\pi}{2}\right]$ for

$81^{\sin^2 x} + 81^{\cos^2 x} = 9$

(1) Zero (2) Two

(3) Three (4) One

Sol. Answer (1)

$81^{\sin^2 x} + 81^{1-\sin^2 x} = 9$

$(81^{\sin^2 x})^2 - 9\,(81)^{\sin^2 x} + 81 = 0$

as $\Delta < 0$, no. solution

14. The coefficient of x^{20} in $(1 + x)(1 + 2x)(1 + 4x)(1 + 8x) \ldots (1 + 2^{20}x)$ is

(1) $2^{211} - 2^{190}$ (2) $2^{191} - 2^{171}$

(3) $2^{231} - 2^{209}$ (4) $2^{161} - 2^{142}$

Sol. Answer (1)

Given expression $= (x + 1)(2x + 1)(2^2x + 1) \ldots (2^{20}x + 1)$

$$= 2^{210}(x+1)\left(x+\frac{1}{2}\right)\left(x+\frac{1}{2^2}\right)\cdots\left(x+\frac{1}{2^{20}}\right)$$

So coeff. of x^{20}

$$= 2^{210}\left[1+\frac{1}{2}+\frac{1}{2^2}+\cdots+\frac{1}{2^{20}}\right]$$

$$= 2^{210}\cdot 1\frac{\left[1-\left(\frac{1}{2}\right)^{21}\right]}{\left(1-\frac{1}{2}\right)}$$

$$= 2^{211}\left(1-\frac{1}{2^{21}}\right) = 2^{211} - 2^{190}$$

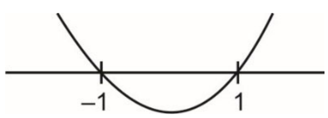
15. Given, $f(x) = \dfrac{x^2-1}{x^2+1}$, find minimum value of $f(x)$

(1) 0 (2) 1

(3) –1 (4) 2

Sol. Answer (3)

$$f(x) = \frac{x^2-1}{x^2+1} = y$$



$$\Rightarrow \frac{2x^2}{2} = \frac{1+y}{1-y}$$

$$\Rightarrow x^2 = \frac{y+1}{1-y} \geq 0$$

$$\Rightarrow \frac{y+1}{y-1} \leq 0$$

$$\Rightarrow y \in [-1, 1)$$

So min $f(x)$ = –1

16. For a binomial probability distribution (33, p) 3P(x = 0) = P(x = 1) then find $\dfrac{P(x=15)}{P(x-18)} - \dfrac{P(x=16)}{P(x=17)}$ is

(1) 1000 (2) 1320

(3) 1221 (4) 1121

Sol. Answer (2)

$x \sim B(33, p)$

$\Rightarrow$ n = 33 and probability of success p

Now $3P(x = 0) = P(x = 1)$

$$3\cdot {}^{33}C_0(1-p)^{33} = {}^{33}C_1\cdot p(1-p)^{32}$$

$\Rightarrow$ $3(1 - p) = 33p$

$$\Rightarrow \boxed{p = -\frac{1}{12}}$$

$$\frac{P(x=15)}{P(x=18)} - \frac{P(x=16)}{P(x=17)}$$

$$= \frac{{}^{33}C_{15}\left(\frac{1}{12}\right)^{15}\left(\frac{11}{12}\right)^{18}}{{}^{33}C_{18}\left(\frac{1}{12}\right)^{18}\left(\frac{11}{12}\right)^{15}} - \frac{{}^{33}C_{16}\left(\frac{1}{12}\right)^{16}\left(\frac{11}{12}\right)^{17}}{{}^{33}C_{17}\left(\frac{1}{12}\right)^{17}\left(\frac{11}{12}\right)^{16}}$$

$$= \left(\frac{11}{12}\right)^3 = 12^3 - \left(\frac{11}{12}\right)\cdot 12$$

$= 11^3 – 11 = 1331 – 11 = 1320$

17. $S = \{\theta : \theta \in [-\pi, \pi] - \left\{\pm\frac{\pi}{2}\right\}$ and sin θ tan θ + tan θ = sin 2θ}

Let $T = \Sigma \cos 2\theta$ where $\theta \in S$, then $T + n(s)$ =

(1) 6 (2) 7

(3) 9 (4) 8

Sol. Answer (3)

$(\sin\theta + 0\ \tan\theta = 2\sin\theta\cos\theta = 0, \theta \in [-\pi, \pi]$ and

$$\theta \neq \pm\frac{\pi}{2}$$

$\sin\theta = 0 \Rightarrow \theta = 0, \pm\pi$

and $\sin\theta + 1 - 2(1-\sin^2\theta) = 0$

$2\sin^2\theta + \sin\theta - 1 = 0$

$$\sin\theta = -1, \sin\theta = \frac{1}{2} \Rightarrow \theta = \frac{\pi}{6}, \frac{5\pi}{6}$$

$$\theta = \frac{-\pi}{2} \text{ not possible}$$

∴ no. of solution = 5

$T = 2\cos 2\theta$

$$= \cos 0 + (\cos 2\pi + \cos(-2\pi) + \cos\frac{\pi}{3} + \cos\frac{5\pi}{3}$$

$$= 3 + \frac{1}{2} + \frac{1}{2} = 4$$

$= T + n(s) = 4 + 5 = 9$

18. A circle of equation $x^2 + y^2 + ax + by + c = 0$, passes through (0, 6) and touches $y = x^2$ at (2, 4). Find $a + c$.

(1) 17 (2) 15

(3) 19 (4) 16

Sol. Answer (4)

Equation of tangent to $y = x^2$ at (2, θ) is

$$\frac{1}{2}(y+4)=2x \Rightarrow 4x-y-4=0$$

Given circle can be written as

$$(x-2)^2+(y-4)^2+\lambda(4x-y-4)=0 \quad \ldots(1)$$

Sub $(0, 6) \Rightarrow 4+4+\lambda(-(0)=0 \Rightarrow \lambda=\frac{4}{5}$

(1) $\Rightarrow$

$$x^2+y^2-4x-8y+20+\frac{4}{5}(4x-y-4)=0$$

$$\Rightarrow \quad x^2+y^2-\frac{4}{5}x-\frac{44}{5}y+\frac{84}{5}=0$$

Comparing with $x^2 + y^2 + ax + by + c = 0$

$a + c = 80 = 16$

19. The equation of plane passing through the line of intersection of planes $x + 2y + 32 = 2$ and $x - y + z = 3$ and at a distance $\frac{2}{\sqrt{3}}$ from (3, 1, – 1) is

(1) $5x + 11y - z = 17 = 0$

(2) $5x + 11y - z - 17 = 0$

(3) $5x - 11y + z + 17 = 0$

(4) $5x - 11y + z - 17 = 0$

Sol. Answer (4)

Equation res plane is $(x + 2y + 3z - 2) + \lambda(x - y + z - 3) = 0$ …(1)

$(1 + \lambda)x + (2 - \lambda)y + (3 + \lambda)z - (2 + 3\lambda) = 0$

dist from (3, 1, –1) = $\frac{2}{\sqrt{3}}$

$$\Rightarrow \left|\frac{3+3\lambda+2-\lambda-3-\lambda-2-3\lambda}{\sqrt{(\lambda-1)^2+(2-\lambda)^2+(3+\lambda)^2}}\right|=\frac{2}{\sqrt{3}}$$

$$\Rightarrow 3\lambda^2=3\lambda^2+4\lambda+14 \Rightarrow \lambda=-\frac{7}{2} \text{ sub in (1)}$$

$-5x + 11y - z + 17 = 0$

$5x - 11y + z - 17 = 0$

20. If the sides of a triangle and $x^2 + x + 1$, $x^2 - 1$, $2x + 1$, find the greatest angle 7 the triangle.

(1) 72° (2) 104°

(3) 120° (4) 108°

Sol. Answer (3)

Sides $x^2 + x + 1 > x^2 - 1 > 2x + 1$ ($\because x > 1$, as $x^2 - 1 > 0$)

$$\cos A=\frac{(x^2-1)^2+(2x+1)^2-(x^2+x+1)^2}{2(x^2-1)(2x+1)}$$

$$=\frac{x^4-2x^2+1+4x^2+4x+1-(x^4+x^2+1+2x^3+2x^2+2x}{2(x^2-1)(2x+1)}$$

$$=\frac{-2x^3-x^2+2x+1}{2(x^2-1)(2x+1)}=\frac{-(x^2-1)(2x+1)}{2(x^2-1)(2x+1)}=\frac{-1}{2}$$

$A = 120°$

21. A and B are two 3 × 3 matrices such that $AB = BA$ then

S-I: if A^3 is symmetric and B^2 is skew symmetric matrix, then $(AB)^6$ is a skew symmetric matrix.

S-II: If A^3 is a skew-symmetric and B^2 is symmetric, then $(AB)^6$ is symmetric.

(1) S-I is true and S-II is false

(2) S-I and S-II both are true

(3) S-I and S-II both are false

(4) S-I is false and S-II is true

Sol. Answer (2)

Statement-I: $(A^3)^T = A^3$, $(B^2)T = B^2$

$(AB)^6 = A^6B^6 = (A^3)^2(B^2)^3$

$$\left[(AB)^6\right]^T=\left[(B^2)^T\right]^3\left[(A^3)^T\right]^2=(-B^2)^3(A^3)^2$$

$= -B^6A^6 = -(BA)^6 = -(AB)^6$

$\therefore$ It is skew symmetric

Statement-II: $(A^3)^T = -A^3$ and $(B^2)^T = B^2$

$$\left[(AB)^6\right]^T=\left[(A^3)^2(B^2)^3\right]^T\left((B^2)^T\right)^3\left((A^3)^T\right)^2$$

$$(B^2)^3(-A^3)^2=(BA)^6=(AB)^6,$$

22. Image of $A\left(\frac{3}{\sqrt{a}}, \sqrt{a}\right)$ in y-axis is B and image of B in x-axis is C. Point D ($3\cos\theta$, $a\sin\theta$) lies in 4th quadrant and the maximum area of ΔACD = 12 sq. units. Then find a.

Sol. Answer (a =8)

$$A=\left(\frac{3}{\sqrt{a}}, \sqrt{a}\right)$$

$$B=\left(-\frac{3}{\sqrt{a}}, \sqrt{a}\right), C=\left(\frac{-3}{\sqrt{a}}, -\sqrt{a}\right)$$

$\Delta = (\cos\theta, a\sin\theta)$

$$\text{Area of } \Delta ACD = \frac{1}{2}\left\|\begin{matrix} \frac{3}{\sqrt{a}} & \sqrt{a} & 1 \\ \frac{-3}{\sqrt{a}} & -\sqrt{a} & 1 \\ 3\cos\theta & a\sin\theta & 1 \end{matrix}\right\|$$

$$R_1 \to R_1 + R_2 = \frac{1}{2}\left\|\begin{matrix} 0 & 0 & 2 \\ \frac{-3}{\sqrt{a}} & -\sqrt{a} & 1 \\ 3\cos\theta & a\sin\theta & 1 \end{matrix}\right\|$$

$$= \left|-3\sqrt{a}\sin\theta + 3\sqrt{a}\cos\theta\right|$$

$$= 3\sqrt{a}\left|\cos\theta - \sin\theta\right| \le 3\sqrt{a} \times \sqrt{2}$$

Max Area $= 3\sqrt{2a} = 12 \Rightarrow 2a = 16$

$a = 8$

23. If $\vec{a}$ and $\vec{b}$ are unit vectors and vectors $\vec{c}$ and $\vec{a}$ have and angle of $\frac{\pi}{12}$ between them such that $\vec{b} = \vec{c} + 2(\vec{c} \times \vec{a})$ then the value of $|6\vec{c}|^2$ is

(1) $3+\sqrt{3}$ (2) $3-\sqrt{3}$

(3) $2+\sqrt{3}$ (4) $2-\sqrt{3}$

Sol. Answer (1)

$|\vec{a}| = |\vec{b}| = 1 \quad (\vec{c}, \vec{a}) = \frac{\pi}{12}$

Given, $\vec{b} - \vec{c} = 2(\vec{c} \times \vec{a}) \quad \ldots(1)$

$(1) \cdot \vec{c} \Rightarrow \vec{b} \cdot \vec{c} = |\vec{c}|^2$

$$|\vec{b} - \vec{c}|^2 = 4|\vec{c} \times \vec{a}|^2 \Rightarrow 1 + c^2 - 2c^2 = 4c^2\left(\frac{\sqrt{3}-1}{2\sqrt{2}}\right)^2$$

$$\Rightarrow 2(1-c^2) = c^2(4-2\sqrt{3})$$

$$\Rightarrow 2 = c^2(6-2\sqrt{3}) \Rightarrow c^2 = \frac{1}{3-\sqrt{3}} = \frac{3+\sqrt{3}}{6}$$

$$6c^2 = 3+\sqrt{3}$$

24. A balloon, spherical in shape is inflated and its surface are a is increasing with a constant rate, initially the radius is 3 units, after 5 sec radius is 7 units then the radius a after 9 second is

(1) 9 (2) 7

(3) 5 (4) 3

Sol. Answer (1)

Given $\frac{ds}{dt} = k$

$$\frac{d}{dt}(4\pi r^2) = k \Rightarrow 8\pi r\frac{dr}{dt} = k$$

$$\Rightarrow \int 8\pi r\,dr = \int k\,dt$$

$$\Rightarrow 4\pi r^2 = kt + c$$

at $t = 0$, $r = 3 \Rightarrow c = 36p$

at $t = 5$, $r = 7 \Rightarrow 196p = 5k + 36p$

$\Rightarrow k = 32\pi$

$4\pi r^2 = 32\pi t + 36\pi \Rightarrow r^2 = 8t + 9$

at $t = 9$, $r^2 = 81 \Rightarrow r = 9$

25. If $x(y) = x$ and $y\frac{dx}{dy} = 2x + y^3(y+1)e^y$ and $x(1) = 0$, then $x(e)$ is equal to

(1) $(e^e - 1)$ (2) $e^3(e^e - 1)$

(3) $3^e - 1$ (4) $e^e - 3$

Sol. Answer (2)

$$y\frac{dx}{dy} = 2x + y^3(y+1)e^y$$

$$\Rightarrow \frac{dx}{dy} + \left(-\frac{2}{y}\right)x = y^2(y+1)e^y$$

$$P = -\frac{2}{y} \quad \theta = y^2(y+1)e^y$$

$$\text{I.F.} = e^{\int -\frac{2}{y}dy} = \frac{1}{y^2}$$

Solution is

$$\frac{x}{y^2} = \int \frac{1}{y^2} \cdot y^2(y+1)e^y dy + c$$

$$\frac{x}{y^2} = ye^y + c \quad x(1) = 0 \Rightarrow c = -e$$

$$\Rightarrow x = y^3e^y - ey^2$$

$$x(1) = e^3 \cdot e^e - e^3 = e^3(e^3 - 1)$$

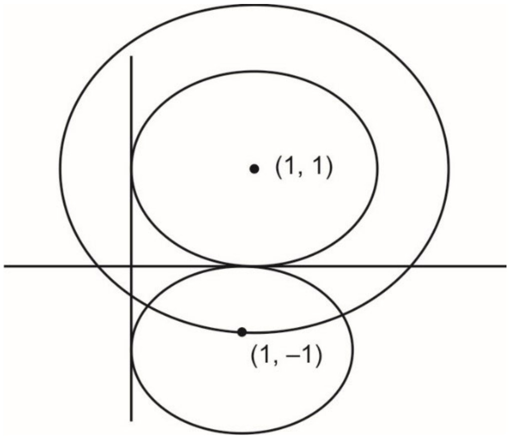
26. If $S = \{z \in C: 1 \le |z - (1+i)| \le 2\}$ and $A = \{z \in s : |z - (1 - I)| = 1\}$

(1) null set (2) singleton set

(3) exactly two element (4) infinite elements

Sol. Answer (4)

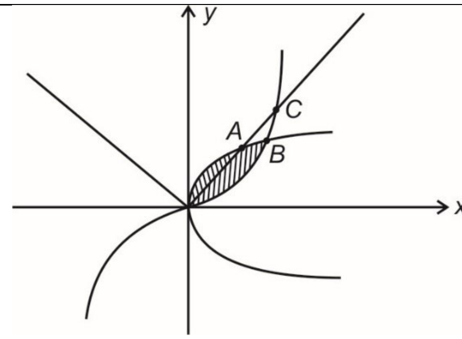
$1 \le |z - (1+i)| \le 2 \qquad |z - (1-i)| = 1$



There are infinitely many point on the circle $|z-(1-i)|=1$, which satisfy $1 \le |z-(1+i)| \le 2$

27. $y = 2|x|$ divides the area enclosed by $y = x^3$ and $y^2 = x$ into two parts of areas R_1 and R_2 where $R_2 > R_1$. Value of $\left(\frac{R_2}{R_1}\right)$ is

Sol. Answer (19)



$y = 2|x|,\ y = x^3,\ y^2 = x$

Solving $y = x^3,\ y^2 = x$

(0, 0) and B = (1, 1)

Solving $y = 2|x|$ and $y^2 = x$

(0, 0) and $A = \left(\frac{1}{4}, \frac{1}{2}\right)$

$$R_1 = \int_0^{1/4} \sqrt{x} - 2x dx = \frac{2}{3} \times \frac{1}{4} \times \frac{1}{2} = \frac{1}{16} = \frac{4-3}{48} = \frac{1}{48}$$

$$R_2 = \int_0^{1/4} 2x - x^3 dr + \int_{1/4}^{1} \sqrt{x} - x^3 dx$$

$$= \frac{1}{16} - \frac{1}{1024} + \frac{2}{3}\left(1 - \frac{1}{8}\right) - \frac{1}{4} + \frac{1}{1024}$$

$$= \frac{1}{16} + \frac{7}{12} - \frac{1}{4} = \frac{3+28-12}{48} = \frac{19}{48} \Rightarrow \frac{R}{R_1} = 19$$

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