



Mathematics JEE Solutions 2022

Mathematics

1. $2 \sin(12^\circ) - \sin(72^\circ) =$

(1) $\frac{1-\sqrt{5}}{8}$

(2) $\frac{\sqrt{5}(1-\sqrt{3})}{4}$

(3) $\frac{\sqrt{3}(1-\sqrt{5})}{2}$

(4) $\frac{\sqrt{3}(1-\sqrt{5})}{4}$

Sol. Answer (4)

$$\begin{aligned} 2 \sin 12^\circ - \sin 72^\circ &= \sin 12^\circ + \sin 12^\circ - \sin 72^\circ \\ &= \sin 12^\circ - 2 \cos 42^\circ \cdot \sin 30^\circ \\ &= -(\cos 42^\circ - \sin 12^\circ) = -(\sin 48^\circ - \sin 12^\circ) \\ &= -2 \cos 30^\circ \sin 18^\circ = -2 \cdot \frac{\sqrt{3}}{2} \left(\frac{\sqrt{5}-1}{4} \right) \\ &= \frac{\sqrt{3}(1-\sqrt{5})}{4} \end{aligned}$$

2. $y = y(x)$ is the solution of the differential equation $2x^2 \frac{dy}{dx} - 2xy + 8y^2 = 0$, $y(e) = \frac{e}{3}$, then $y(1)$ is equal to

(1) $\frac{2}{3}$

(2) 3

(3) $\frac{3}{2}$

(4) -1

Sol. Answer (4)

$$2x^2 \frac{dy}{dx} - 2xy + 8y^2 = 0$$

$$\frac{dy}{dx} = \frac{xy - 4y^2}{x^2}, y = vx$$

$$v + \frac{xdv}{dx} = v - 4v^2 \Rightarrow \int \frac{dv}{v^2} = -4 \int \frac{dx}{x}$$

$$\Rightarrow -\frac{1}{v} = -4 \ln x - \ln c \Rightarrow \frac{x}{y} = \ln cx^4$$

$$x = e, y = \frac{e}{3} \Rightarrow 3 = \ln c + 4 \Rightarrow \ln c = -1 \Rightarrow c = e^{-1}$$

$$\frac{x}{y} = \ln \frac{x^4}{e}$$

$$x = 1 \Rightarrow \frac{1}{y} = \ln \frac{1}{e} = -\ln e = -1 \Rightarrow y = -1$$

3. $A = \{x \in R : |x+1| < 2\}$, $B = \{x \in R : |x-1| \geq 2\}$
then

(1) $A \cup B = R - [1, 3]$

(2) $A \cap B = [-1, 1]$

(3) $A \cap B = (-3, -1]$

(4) $B - A = R - (-3, 1]$

Sol. Answer (3)

$$|x+1| < 2 \Rightarrow x+1 \in (-2, 2) \Rightarrow x \in (-3, 1) = A$$

$$|x-1| \geq 2 \Rightarrow x-1 \leq -2 \text{ or } x-1 \geq 2$$

$$\Rightarrow x \leq -1 \text{ or } x \geq 3$$

$$\Rightarrow x \in (-\infty, -1] \cup [3, \infty) = B$$

$$A \cup B = (-\infty, 1) \cup [3, \infty) = R - [1, 3]$$

$$A \cap B = (-3, -1]$$

$$B - A = (-\infty, -3] \cup [3, \infty) = R - (-3, 3)$$

4. Find the sum $S = 1 + 2.3 + 3.3^2 + \dots + 10.3^9$

(1) $\frac{1}{4}(19.3^{10} + 1)$

(2) $\frac{1}{4}(19.3^{10} - 1)$

(3) $\frac{1}{2}(19.3^{10} + 1)$

(4) $\frac{1}{2}(19.3^{10} - 1)$

Sol. Answer (1)

$$S = 1 + 2.3 + 3.3^2 + \dots + 10 \times 10 \times 3^9$$

$$3S = 0 + 3 + 2.3^2 + \dots + 9 \times 3^9 + 10 \times 3^{10}$$

Subtracting $-2s = 1 + 3 + 3^2 + \dots + 3^9 - 10 \times 3^{10}$

$$-2s = \frac{3^{10} - 1}{3 - 1} - 10 \times 3^{10}$$

$$\Rightarrow s = 5 \times 3^{10} - \frac{(3^{10} - 1)}{4} = \frac{19 \cdot 3^{10} + 1}{4}$$

5. Water increasing in a right circular cone with rate $1 \text{ cm}^3/\text{sec}$. then rate change of lateral surface of cone is (where height and diameter of cone is 35 and 14 respectively) at $h = 10 \text{ cm}$.

(1) $\frac{\sqrt{26}}{10}$ (2) $\frac{\sqrt{26}}{5}$

(3) $\frac{\sqrt{21}}{5}$ (4) 5

Sol. Answer (2)

$$\frac{h}{r} = \frac{35}{7} \Rightarrow h = 5r$$

$$v = \frac{1}{3} \pi r^2 h = \frac{1}{3} \pi r^2 \cdot 5r = \frac{5}{3} \pi r^3$$

$$\frac{dv}{dt} = \frac{5}{3} \pi \cdot 3r^2 \frac{dr}{dt} = 1 \Rightarrow \frac{dr}{dt} = \frac{1}{5\pi r^2}$$

$$\text{Now } s = \pi r l = \pi r \sqrt{r^2 + h^2}$$

$$= \pi r \sqrt{r^2 + 25r^2} = \pi r^2 \sqrt{26}$$

$$\frac{ds}{dt} = 2\pi r \cdot \frac{dr}{dt} \cdot \sqrt{26} = \frac{2\pi r}{5\pi r^2} \cdot \sqrt{26} = \frac{2 \cdot \sqrt{26}}{5r}$$

$$\text{when } h = 10 \Rightarrow r = 2$$

$$\frac{ds}{dt} = \frac{\sqrt{26}}{5}$$

6. 2, 4, 8, 16, 32, 32 written on a faces of a biased dice probability of showing up face with mark n is $\frac{1}{n}$ of dice is rolled 3 times. Find probability of sum coming as 48.

(1) $\frac{7}{2^{12}}$ (2) $\frac{3}{2^{10}}$

(3) $\frac{7}{2^{11}}$ (4) $\frac{13}{2^{12}}$

Sol. Answer (4)

$$P(\text{sum} = 48) = P(16) \times P(16) \times P(16) + P(8) \times P(32) \times P(8)$$

$$= \frac{1}{16} \times \frac{1}{16} \times \frac{1}{16} + {}^3C_1 \times \left(\frac{1}{8}\right)^2 \times \left(\frac{2}{32}\right)$$

$$= \frac{1+12}{16^3} = \frac{13}{16^3} = \frac{13}{2^{12}}$$

7. In the series

$$(5+x)^{500} + x(5+x)^{499} + \dots + x^{500}$$

find coefficient of x^{101}

- (1) 5^{300} (2) 5^{399}
(3) $5^{399} {}^{501}C_{101}$ (4) $-5^{399} {}^{501}C_{101}$

Sol. Answer (3)

$$\frac{(5+x)^{500} \left(\left(\frac{x}{5+x} \right)^{501} - 1 \right)}{\left(\frac{x}{5+x} \right) - 1} = \frac{x^{501} - (5+x)^{501}}{(-5)}$$

$$= \frac{1}{5} (5+x)^{501} - \frac{x^{501}}{5}$$

for coefficient of x^{101} , consider only $\frac{1}{5} (5+x)^{501}$

$$\therefore \frac{1}{5} ({}^{501}C_{101} \cdot 5^{400} \cdot x^{101})$$

$$\Rightarrow \text{coefficient of } x^{101} \text{ is } 5^{399} \cdot {}^{501}C_{101}$$

8. Negation of the statement $(\sim p \vee q) \Rightarrow (\sim q \wedge p)$

- (1) $\sim p \Rightarrow q$ (2) $p \Rightarrow q$
(3) $\sim q \Rightarrow p$ (4) $p \Leftrightarrow q$

Sol. Answer (2)

$$(\sim p \vee q) \Rightarrow (\sim q \wedge p) \equiv \sim(\sim p \vee q) \vee (p \wedge \sim q)$$

$$\equiv (\sim q \wedge p) \vee (\sim q \wedge p) \equiv (p \wedge \sim q)$$

$$\text{Now } \sim[(\sim p \vee q) \Rightarrow (\sim q \wedge p)] \equiv \sim(p \wedge \sim q)$$

$$\sim[(\sim p \vee q) \Rightarrow (\sim q \wedge p)] \equiv \sim(p \wedge \sim q)$$

$$\equiv \sim p \vee q$$

$$\equiv p \Rightarrow q$$

9. If $b_n = \int_0^{\pi/2} \frac{\cos^2 nx}{\sin x} dx$ then

- (1) $\frac{1}{b_3 - b_2}, \frac{1}{b_4 - b_3}, \frac{1}{b_5 - b_4}$ are in A.P. with common difference = -2

- (2) $\frac{1}{b_3 - b_2}, \frac{1}{b_4 - b_3}, \frac{1}{b_5 - b_4}$ are in A.P. with common difference = 2
- (3) $b_3 - b_2, b_4 - b_3, b_5 - b_4$ are in G.P.
- (4) $b_3 - b_2, b_4 - b_3, b_5 - b_4$ are in A.P. with common difference = -2

Sol. Answer (1)

$$b_n - b_{n-1} = \int_0^{\pi/2} \frac{\cos^2 nx - \cos^2(n-1)x}{\sin x} dx$$

$$= \int_0^{\pi/2} \frac{\sin(2n-1)x(-\sin x)}{\sin x} dx$$

$$= \frac{-1}{2n-1}$$

$$n = 3, b_3 - b_2 = \frac{-1}{5}$$

$$n = 4, b_4 - b_3 = \frac{-1}{7}$$

$$n = 5, b_5 - b_4 = \frac{-1}{9}$$

$$\therefore \frac{1}{b_3 - b_2}, \frac{1}{b_4 - b_3}, \frac{1}{b_5 - b_4}$$

are in A.P. with common difference is -2

- 10.** Equation of tangent at p on parabola $y = x - x^2$ is $y = 4 + kx$, $x > 0$ and let v be the vertex of parabola, then slope of line joining p and v is

- (1) $-\frac{3}{2}$ (2) $\frac{26}{9}$
- (3) $\frac{5}{2}$ (4) $\frac{23}{6}$

Sol. Answer (2)

$$(x+1)y^1 = y = e^{3x}(x+1)^2$$

$$y^2 - \frac{1}{x+1}y = e^{3x}(x+1) \quad y(0) = \frac{1}{3}$$

$$\text{I.F.} = e^{\int \frac{-1}{x+1} dx} = e^{-\log(x+1)}$$

$$\text{so solution is } y \cdot \frac{1}{x+1} = \int e^{3x} dx + c$$

$$\frac{y}{x+1} = \frac{e^{3x}}{3} + c$$

$$x = 0 \Rightarrow \frac{1}{3} = \frac{1}{3} + c \Rightarrow c = 0$$

$$y = \frac{(x+1)}{3} e^{3x}$$

$$x = \frac{-4}{3} \Rightarrow y = \frac{-1}{9e^4}$$

- 11.** If system of equation

$$-kx + 3y - 14z = 25$$

$$-15x + 4y - kz = 3$$

$$-4x + y + 3z = 4$$

consistent then k belongs

- (1) $R - \{-11, 13\}$ (2) $R - \{-11, 11\}$
- (3) $R - \{13\}$ (4) for all $k \in R$

Sol. Answer (2)

$$\Delta = \begin{vmatrix} -k & 3 & -14 \\ -15 & 4 & -k \\ -4 & 1 & 3 \end{vmatrix} \neq 0$$

$$\Rightarrow -k(12+k) - 3(-45-4k) - 14(-15+16) \neq 0$$

$$\Rightarrow -12k - k^2 + 135 + 12k - 14 \neq 0$$

$$k^2 \neq 121 \Rightarrow k \neq \pm 11$$

$$k \in R - \{-11, 11\}$$

- 12.** Find the value of $\tan^{-1} \left[\frac{\cos\left(\frac{15\pi}{4}\right) - 1}{\sin \frac{\pi}{4}} \right]$

- (1) $-\frac{\pi}{8}$ (2) $-\frac{4\pi}{9}$
- (3) $-\frac{5\pi}{12}$ (4) $-\frac{\pi}{4}$

Sol. Answer (1)

$$\tan^{-1} \left(\frac{\cos \frac{15\pi}{4}}{\sin \frac{\pi}{4}} \right) = \tan^{-1} \left(\frac{\cos \left(4\pi - \frac{\pi}{4} \right) - 1}{\sin \frac{\pi}{4}} \right)$$

$$= \tan^{-1} \left(\frac{\cos \frac{\pi}{4} - 1}{\sin \frac{\pi}{4}} \right) = \tan^{-1} \left(\frac{-2 \sin^2 \frac{\pi}{8}}{2 \sin \frac{\pi}{8} \cos \frac{\pi}{8}} \right)$$

$$= \tan^{-1} \left(\tan \frac{-\pi}{8} \right) = -\frac{\pi}{8}$$

- 13.** $ax^2 - 2bx + 15 = 0$, $(a, b \in R)$ α is a repeated root and $x^2 - 2bx + 21 = 0$ have roots α and β then $\alpha^2 + \beta^2 =$

(1) 58 (2) 56

(3) 57 (4) 60

Sol. Answer (1) α and β are roots of $x^2 - 2bx + 21 = 0$

$$\Rightarrow \alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta$$

$$= 4b^2 - 2 \times 21 = 4b^2 - 42$$

Further, α is repeated root of $ax^2 - 2bx + 15 = 0$

$$\Rightarrow \alpha \cdot \alpha = \frac{15}{a} \text{ and } 2\alpha = \frac{2b}{a}$$

$$\Rightarrow \alpha^2 = \frac{15}{a} \text{ and } \boxed{\alpha = \frac{b}{a}}$$

Now, substituting α in $x^2 - 2bx + 21 = 0$

$$\Rightarrow \frac{b^2}{a^2} - \frac{2b^2}{a} + 21 = 0$$

$$\Rightarrow b^2 - 2b^2a + 21a^2 = 0$$

$$\Rightarrow 15a - 30a^2 + 12a^2 = 0$$

$$\Rightarrow 15a = 9a^2$$

$$\Rightarrow a = 0 \text{ (Not possible) or } a = \frac{5}{3}$$

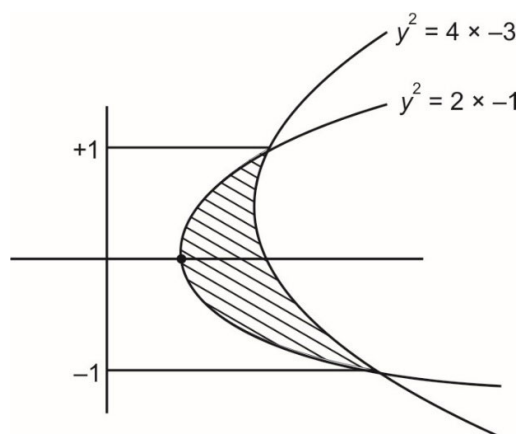
$$\therefore \alpha^2 + \beta^2 = 4b^2 - 42 = 4 \times 15a - 42$$

$$= 60 \times \frac{5}{3} - 42 = 100 - 42 = 58$$

14. Area of the region bounded by the curves $y^2 = 2x - 1$ and $y^2 = 4x - 3$ is

(1) $\frac{1}{6}$ (2) $\frac{1}{3}$

(3) $\frac{2}{3}$ (4) $\frac{1}{2}$



$$y^2 = 2x - 1$$

$$y^2 = 4x - 3$$

Point of intersection is $(1, -1)$ and $(1, 1)$

$$\text{Area} = \int_{-1}^{+1} \left[\left(\frac{y^2 + 3}{4} \right) - \left(\frac{y^2 + 1}{2} \right) \right] dy$$

$$\int_{-1}^{+1} \left(\frac{1}{4} - \frac{y^2}{4} \right) dy = 2 \int_0^1 \left(\frac{1}{4} - \frac{y^2}{4} \right) dy$$

$$= 2 \left(\frac{1}{4} - \frac{1}{12} \right) = \frac{1}{3}$$

15. If sum of first n terms of two AP's are in ratio of $3n + 8 : 7n + 15$ then the ratio of their 12th term is

(1) 8 : 7 (2) 7 : 16

(3) 74 : 169 (4) 13 : 47

Sol. Answer (2)

$$\frac{S_n}{S'_n} = \frac{3n + 8}{7n + 15}$$

$$\Rightarrow \frac{2a + (n-1)d}{2A + (n-1)D} = \frac{3n + 8}{7n + 15}$$

$$\text{Put } n = 23 \Rightarrow \frac{a + 11d}{A + 11D} = \frac{77}{176} = \frac{7}{16}$$

$$\frac{t_{12}}{t'_{12}} = \frac{7}{16}$$

16. The value of

$$\lim_{x \rightarrow \frac{\pi}{2}} \tan^2 x \left(\sqrt{2\sin^2 x + 3\sin x + 4} - \sqrt{\sin^2 x + 6\sin x + 2} \right)$$

is

(1) $-\frac{1}{12}$ (2) $\frac{1}{18}$

(3) $\frac{1}{12}$ (4) $-\frac{1}{18}$

Sol. Answer (3)

$$\lim_{x \rightarrow \frac{\pi}{2}} \tan^2 x \left(\sqrt{2\sin^2 x + 3\sin x + 4} - \sqrt{\sin^2 x + 6\sin x + 2} \right)$$

(0 $\times$ ∞ form)

$$\lim_{x \rightarrow \frac{\pi}{2}} \tan^2 x \left(\frac{2\sin^2 x + 3\sin x + 4 - (\sin^2 x + 6\sin x + 2)}{\sqrt{2\sin^2 x + 3\sin x + 4} + \sqrt{\sin^2 x + 6\sin x + 2}} \right)$$

$$\lim_{x \rightarrow \frac{\pi}{2}} \frac{\tan^2 x (\sin^2 x - 3\sin x + 2)}{(3+3)}$$

$$\lim_{x \rightarrow \frac{\pi}{2}} \frac{\sin^2 x}{6 \cos^2 x} (\sin x - 1)(\sin x - 2)$$

$$\lim_{x \rightarrow \frac{\pi}{2}} \frac{1 \times (1-2) (\sin x - 1)}{6 \quad 1 - \sin^2 x}$$

$$= \lim_{x \rightarrow \frac{\pi}{2}} \frac{-1}{6} \times \frac{(-1)}{1 + \sin x}$$

$$= \frac{-1}{6} \times \frac{1}{2} = \frac{1}{12}$$

17. Find the number of 3 digit numbers which has exactly 2 digits identical

Sol. Answer (243)

Case-I: $\frac{a}{a} \frac{a}{b} \frac{b}{a}$

Number of such number = $9 \times 9 \times 2$ (a can not be zero) = 162

Case-II: $\frac{b}{b} \frac{a}{a} \frac{a}{a}$

Number of such number = 9×9 (b can not be zero) = 81

Number of such number $162 + 81 = 243$

18. Mean deviation about mean = $\frac{5(n+1)}{n}$ of data 1, 2, 3, ... n. Where n is odd, then values of n is equal to

Sol. Answer (21)

Mean of 1, 2, ... n = $\frac{n+1}{2} = \frac{2k+1+1}{2} = k+1$,

where $n = 2k + 1$

Mean deviation =

$$\frac{|1 - (k+1)| + |2 - (k+1)| + \dots + |(2k+1) - (k+1)|}{2k+1}$$

$$= \frac{2(1+2+\dots+k)}{2k+1} = \frac{k(k+1)}{2k+1} = \frac{(n-1)(n+1)}{4n}$$

Given $\frac{n^2-1}{4n} = \frac{5(n+1)}{n}$

$$\Rightarrow (n-1)(n+1) = 20(n+1)$$

$$\Rightarrow n = 21$$

19. $12 \int_3^b \frac{dx}{(x^2-4)(x^2-1)} = \ln \frac{49}{50}$, then number of possible values of b are

Sol. Answer (4)

Given $12 \int_3^b \frac{dx}{(x^2-4)(x^2-1)} = \ln \frac{49}{50}$

$$\frac{12}{3} \left(\int_3^b \left(\frac{1}{x^2-4} - \frac{1}{x^2-1} \right) dx \right) = \ln \frac{49}{50}$$

$$\Rightarrow 4 \left(\frac{1}{4} \ln \left| \frac{x-2}{x+2} \right| - \frac{1}{2} \ln \left| \frac{x-1}{x+1} \right| \right) \Big|_3^b = \ln \frac{49}{50}$$

$$\Rightarrow \ln \left| \frac{b-2}{b+2} \right| - \ln \left| \frac{1}{5} \right| - 2 \left(\ln \left| \frac{b-1}{b+1} \right| - \ln \left| \frac{2}{4} \right| \right) = \ln \frac{49}{50}$$

$$\Rightarrow \ln \left(\frac{b-2}{b+2} \times 5 \times \left(\frac{b+1}{b-1} \right)^2 \right) = \ln \frac{49}{50}$$

$$\Rightarrow \frac{(b-2)(b+1)^2}{(b+2)(b-1)^2} = \frac{49 \times 4}{50 \times 5} = \frac{98}{125}$$

$$\Rightarrow \frac{b^3 + 2b^2 + b - 2b^2 - 4b - 2}{b^3 - 2b^2 + b + 2b^2 - 4b + 2} = \frac{98}{125}$$

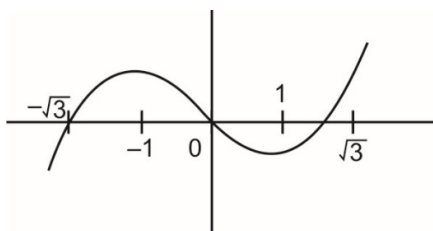
$$\Rightarrow \frac{b^3 - 3b - 2}{b^3 - 3b + 2} = \frac{98}{125}$$

Using componendo and Dividendo,

$$\Rightarrow \frac{b^3 - 3b}{2} = \frac{223}{27} \text{ or } \frac{27}{223}$$

$$\therefore b^3 - 3b = \frac{446}{27} \text{ or } b^3 - 3b = \frac{54}{223}$$

Now, let $f(b) = b^3 - 3b$



$f(b)$ has local maximum at $x = -1$

$$f(-1) = -1 + 3 = 2$$

$$\therefore b^3 - 3b = \frac{444}{27} \text{ has only one solution}$$

$$b^3 - 3b = \frac{54}{223} \text{ has 3 solution}$$

$\therefore$ 4 distinct values of b are possible.

20. If sum of coefficients of positive even powers of x in the expansion of $\left(2x^3 + \frac{3}{x}\right)^{10}$ is $5^{10} - \beta \cdot 3^9$, then value of β is

Sol. Answer (83)

Given Binomial expression $\left(2x^3 + \frac{3}{x}\right)^{10}$

General term =

$${}^{10}C_r (2x^3)^r \left(\frac{3}{x}\right)^{10-r} = {}^{10}C_r 2^r 3^{10-r} x^{3r+r-10}$$

$$= {}^{10}C_r 2^r 3^{10-r} x^{4r-10}$$

$\therefore r$ cannot be 0, 1, 2

$\therefore$ Required sum =

$$5^{10} - \left({}^{10}C_0 2^0 3^{10} + {}^{10}C_1 2^1 3^9 + {}^{10}C_2 2^2 3^8\right)$$

$$= 5^{10} - (3^{10} + 10 \times 2 \times 3^9 + 45 \times 4 \times 3^8)$$

$$= 5^{10} - (3^{10} + 20 \times 3^9 + 60 \times 3^9)$$

$$= 5^{10} - 83 \times 3^9$$

$$\therefore \beta = 83$$

21. Hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ has eccentricity $\frac{5}{4}$. If

normal to the hyperbola at $\left(\frac{8}{\sqrt{5}}, \frac{12}{5}\right)$ is

$8\sqrt{5}x + \beta y = \lambda$. Then the value of $(\lambda - \beta)$ is

Sol. Answer (85)

$$b^2 = a^2 (e^2 - 1)$$

$$\Rightarrow \frac{b^2}{a^2} = \frac{25}{16} - 1 = \frac{9}{16}$$

$$\Rightarrow \frac{b}{a} = \frac{3}{4}$$

Now equation of normal at (x_1, y_1) is

$$\Rightarrow \frac{a^2 x}{x_1} + \frac{b^2 y}{y_1} = a^2 + b^2 = a^2 e^2$$

$$\Rightarrow \frac{a^2 x}{8/\sqrt{5}} + \frac{b^2 y}{12/5} = a^2 \cdot \frac{25}{16} \text{ dividing by } a^2$$

$$\Rightarrow \frac{\sqrt{5}x}{8} + \frac{b^2}{a^2} \cdot \frac{5y}{12} = \frac{25}{16}$$

$$\Rightarrow \frac{\sqrt{5}x}{8} + \frac{9}{16} \cdot \frac{5y}{12} = \frac{25}{16}$$

$$\Rightarrow 8\sqrt{5}x + 15y = 100$$

$$\text{So, } \lambda - \beta = 100 - 15 = 85$$

