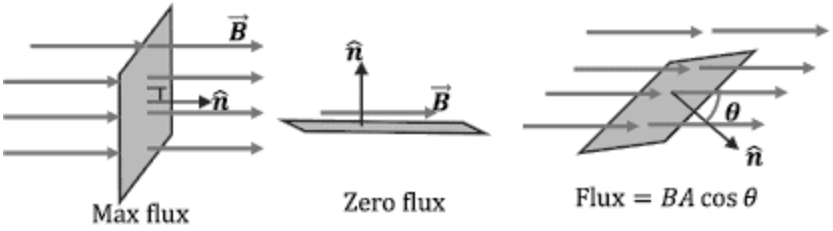
Electromagnetic Induction

Electricity and Magnetism are parallel phenomenon which goes parallelly to each other. Michael Faraday did an experiment where is change magnetic field periodically then he saw there was change in electric filed and hence due to change in electric field electric potential created so current produced simultaneously, this current is called as ***Induced current.*** And, this phenomenon is known as **Electromagnetic Induction**.

Formulae:

Magnetic Flux:

- Magnetic flux through a surface whose area is **A** and magnetic field **B** lines goes through it. Hence, $\phi = B.A = BA\cos\theta$.



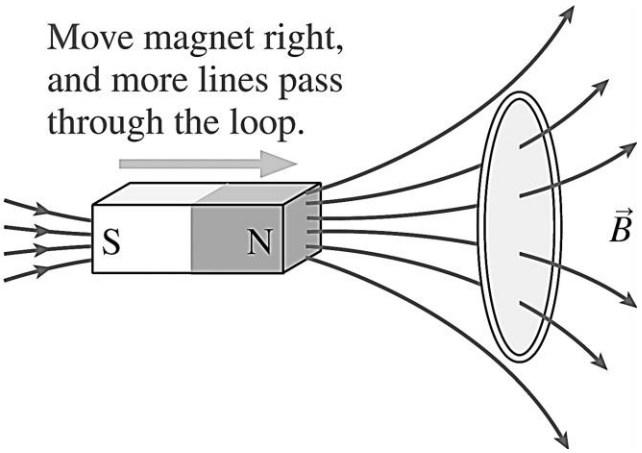
- If the magnetic field at different areas of a surface has varied magnitudes and orientations, then the magnetic flux across the surface is-

$$\phi = B_1.dA_1 + B_2.dA_2 + B_3.dA_3 + ... = \sum_{all} B_i.dA_i$$

Where, 'all' means every point on the surface and SI unit of magnetic flux is Weber **Wb** or **Tm^2** and it is a scalar quantity.

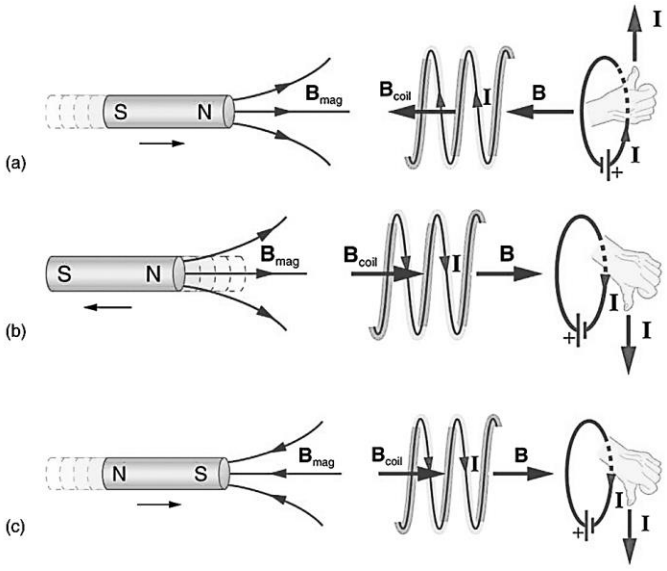
Faraday's Law of Induction:

- He gave idea of induced emf in a circuit which is equal to the time rate of change of magnetic flux. That is- $\varepsilon = -\dfrac{d\phi_B}{dt}$
- Negative sign indicates the polarity of ε and the direction of current in closed loop.
- In case of wound coil which has *N* turns then $\varepsilon = \dfrac{-Nd\phi_B}{dt}$.



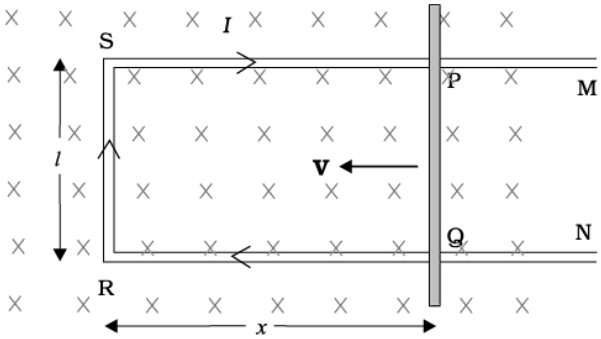
Lenz's Law:

- Lenz's law states that the polarity of induced emf is such that it tends to produce a current which opposes the change in magnetic flux that produced it. Which means,

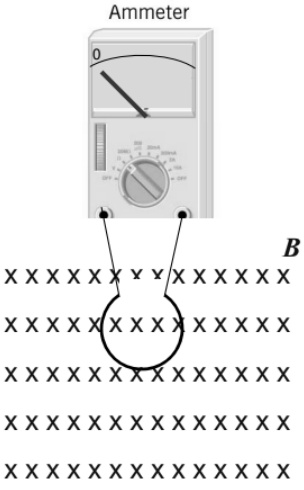


nature "resists" the change in magnetic flux.

Motional Electromotive Force:



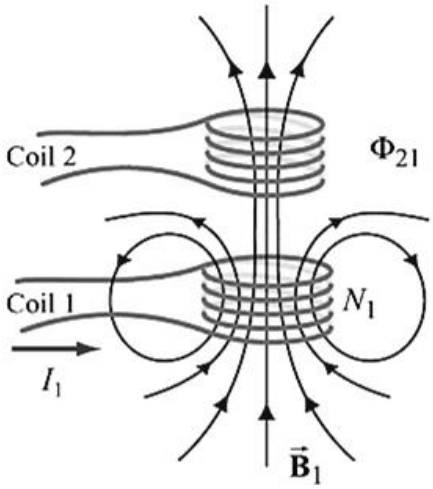
- If a straight conductor moves with uniform speed in time- independent magnetic field then EMF would be: $\varepsilon = -Bl\dfrac{dx}{dt} = Blv$ where $V = \dfrac{-dx}{dt}$
- Work done in moving the charge: $W = q\,v\,B\,\ell$
- The alternative way to produce induced emf and induced current by changing the shape of the coil i.e., by **shrinking of coil or enlarging the coil** and rotate the coil.



Energy Consideration in Motional Emf:

- When r be the resistance of movable arm PQ and then overall resistance of rectangular loop PQRS becomes r. Thus, the current flowing through the loop is $I=\frac{\varepsilon}{r}=\frac{Bvl}{r}$.
- Magnitude of force is $|\vec{F}|=I\ell B=\frac{B^2\ell^2V}{r}$
- The power required to move the arm is $P=F.v=\frac{B^2l^2v^2}{r}$
- According to the law of conservation of energy, the work done on the charges is equal to the energy gained by the electrical system. Therefore, the mechanical energy expended in moving the conductor is converted into electrical energy and then in heat energy.

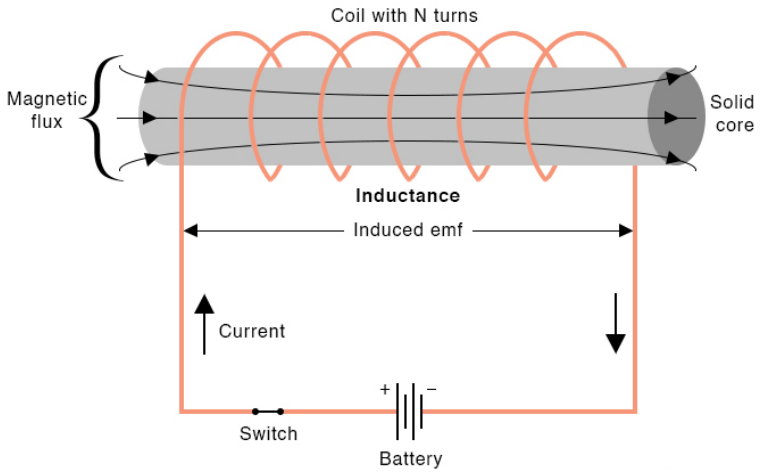
Inductance: An electric current can be induced in a coil by a flux change caused by another coil nearby or by a flux change caused by the same coil. Hence, the flux through a coil is proportional to the current. Thus, $\phi \propto I$. And, $\frac{d\phi}{dt}\propto\frac{dI}{dt}$. For N closely wound coil $N\phi \propto I$. SI unit of inductance is henry **H.**



- **Mutual Inductance:**
  - When current I_2 and I_1 through S_2 and S_1 then $N_1\phi_1 \propto I_2$ then $N_1\phi_1 = M_{12}I_2$
  - where $M_{12}=$ Mutual Inductance of solenoid S_1 with respect to S_2.
  - Hence, $N_1\phi_1 = n_1 l\pi r_1^2(\mu_o n_2 I_2)$.Similarly, $N_2\phi_2 = n_2 l\pi r_1^2(\mu_o n_1 I_1)$
  - $M_{12}=M_{21}=M$ so $M=\mu_r\mu_o n_1 n_2 \pi r_1^2 l$
- **Self-Inductance:**
  - It is also possible that emf is generated in a single isolated coil as a result of a change in flux through the coil caused by altering the current through the same coil.
  - A flux goes through a coil of N turns is directly proportional to the current flowing through the coil: $N\phi \propto I$
  - $N\phi = LI$ where L= self-inductance of the coil.
  - The induced emf is given by: $\varepsilon=-\frac{d(N\phi)}{dt}=-L\frac{dI}{dt}$
  - The total flux linked with solenoid is: $N\phi = nl\mu_o nIA=\mu_o n^2 AlI$
  - Hence, the self-inductance would be: $L=\mu_r\mu_o n^2 Al$
  - The energy corresponds to the inductance is: $W=\frac{1}{2}LI^2$

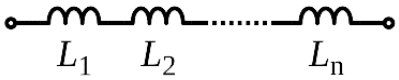
Electromagnetic Induction

- Induced emf in terms of both mutual and self-inductance is: $\varepsilon_1 = -L\frac{dI_1}{dt} - M_{12}\frac{dI_2}{dt}$

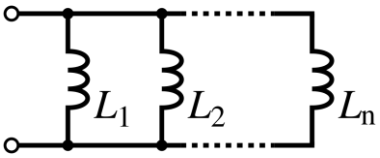


Combination of Inductors:

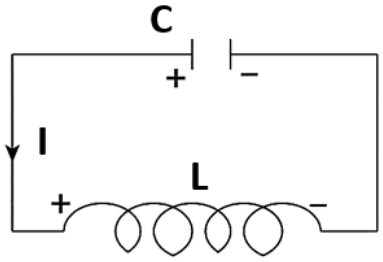
- **Inductors in Series:** When inductors are connected in series, their total inductance can be found using the following formula: $L_{total} = L_1 + L_2 + L_3 + \ldots L_n$



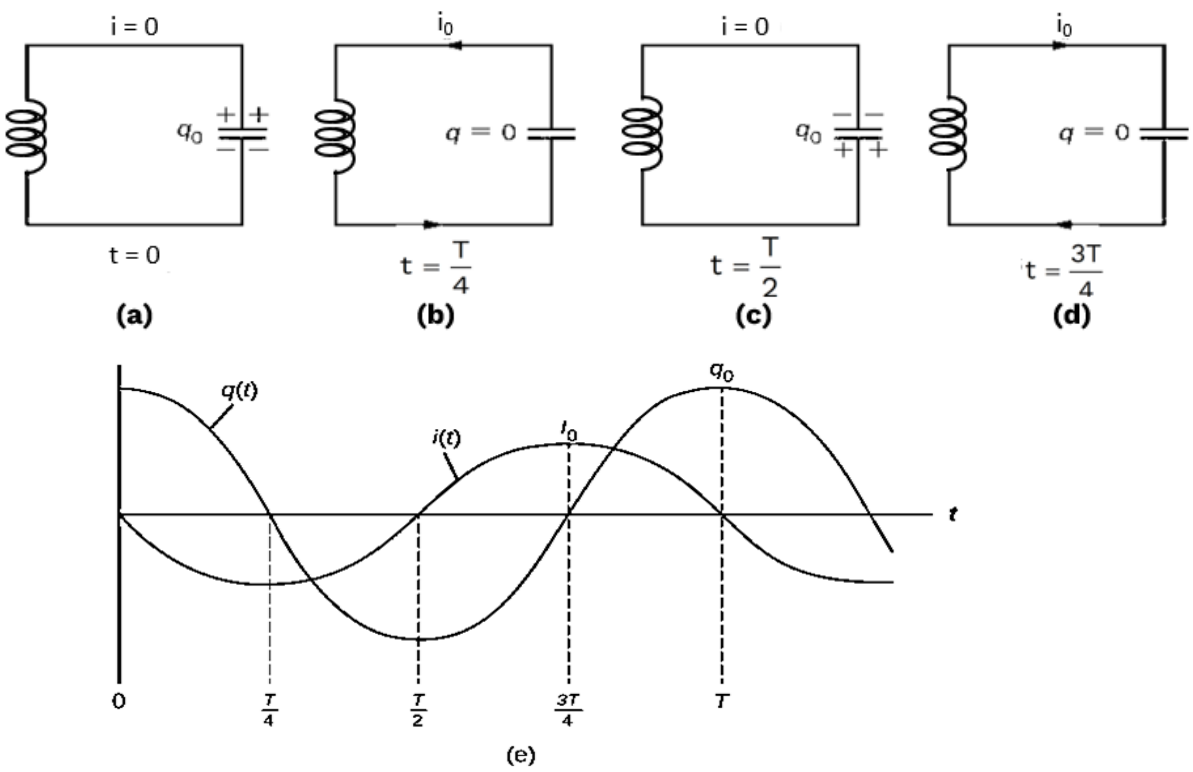
- **Inductors in Parallel:** When inductors are connected in parallel, their total inductance can be found using the reciprocal formula. The formula for calculating the total inductance (L_{total}) of inductors in parallel is as follows: $\frac{1}{L_{total}} = \frac{1}{L_1} + \frac{1}{L_2} + \frac{1}{L_3} + \ldots\ldots \frac{1}{L_n}$



LC Oscillation: LC oscillation refers to the oscillatory behavior in an electrical circuit containing an inductor (L) and a capacitor (C). In such a circuit, energy is exchanged between the electric field stored in the capacitor and the magnetic field stored in the inductor. The interplay of these energy storage elements leads to periodic oscillations in voltage or current, creating a sinusoidal waveform.



- By using Kirchhoff's voltage law on the above circuit, the voltage across capacitor and inductor is zero: $V_L + V_C = 0$
- The current (i) flow through the circuit will be $-L\frac{dI}{dt} + \frac{q}{C} = 0$ and by putting the value of current i.e., $I = \frac{dq}{dt}$ Thus, $\frac{d^2q}{dt^2} + \frac{q}{LC} = 0$
- Angular frequency $\omega = \frac{1}{\sqrt{LC}}$
- Differential equation will become: $\frac{d^2q}{dt^2} + q\omega^2 = 0$
- If $\omega = 2\pi f$ *then* $f = \frac{1}{2\pi\sqrt{LC}}$
- Time period, $T = \frac{1}{f} = 2\pi\sqrt{LC}$
- **Waveform of LC Oscillation:**



AC- Generator:

- AC- Generator is basically a machine which produces alternating current by using induced emf.
- When a coil rotates with a constant angular speed ω then the angle θ between magnetic field B and area A created. We know $\theta = \omega t$. Hence, induced emf would be $\varepsilon = -N\frac{d\phi}{dt} = -NBA\frac{d(\cos\omega t)}{dt}$

- $\varepsilon = NBA\omega \sin \omega t$ and $\varepsilon_o = NBA\omega$ so $\varepsilon = \varepsilon_o \sin \omega t$