Moving Charges and Magnetism

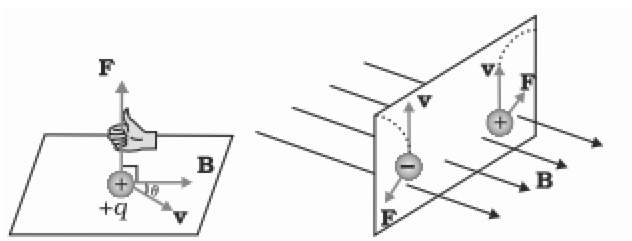
When charge moves it also produces magnetic field. According to Oersted, he concluded that moving charges or currents produced a magnetic field in the surrounding space.

Formulae:

Lorentz Force: Combination of two forces –electrostatic force and magnetic force. Hence,

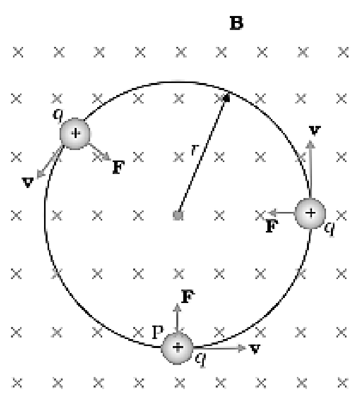
$$\vec{F} = \vec{F}_{Electric} + \vec{F}_{Magnetic} = q[\vec{E} + (\vec{v} \times \vec{B})] = qE + qvB\sin\theta$$

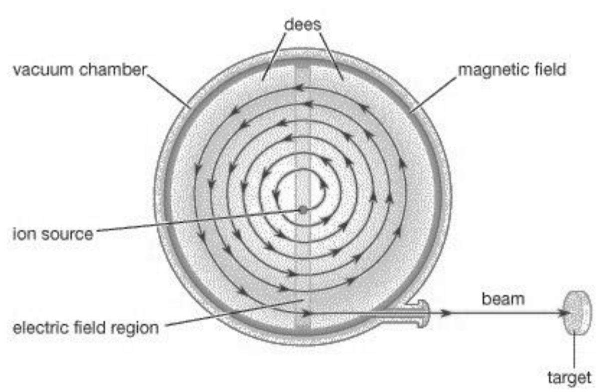
- Magnetic force on a straight current-carrying conductor: $\vec{F} = I(\vec{l} \times \vec{B})$
- For an any shaped wire: $\vec{F} = I\sum_j \vec{dl}_j \times \vec{B}$



Motion in Magnetic Field:

- When moving charge is kept in motion in magnetic field then it follows helical path.





Hence, $\frac{mv^2}{r} = qvB$, $r = \frac{mv}{qB}$

- Angular Frequency, $\omega = 2\pi v = \frac{qB}{m}$

Cyclotron:

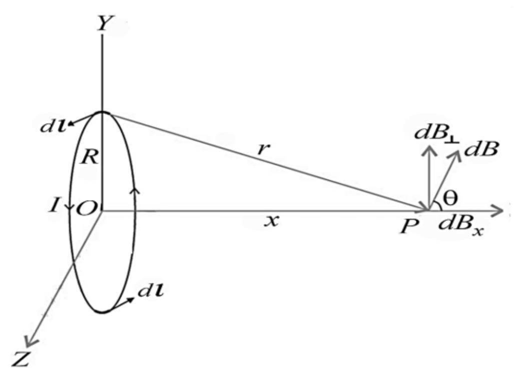
- Cyclotron Frequency, $v_c = \dfrac{qB}{2\pi m}$
- Time Period, $T = \dfrac{2\pi m}{qB}$
- Condition for resonance: Applied voltage frequency(V_a)=Cyclotron Frequency(V_c)
- Velocity of accelerated ion, $v = \dfrac{qBR}{m}$
- Kinetic Energy of ions: $\dfrac{1}{2}mv^2 = \dfrac{q^2B^2R^2}{2m}$

Bio Savart's Law:

- Magnetic field due to current element dl at a distance r, $dB = \dfrac{\mu_o}{4\pi}\dfrac{|\vec{dl}|\times\vec{r}}{r^3}$,

  $|dB| = \dfrac{\mu_o}{4\pi}\dfrac{|dl|\sin\theta}{r^2}$

  where



$\dfrac{\mu_o}{4\pi} = 10^{-7} Tm/A$ and μ_o=permeability of free space

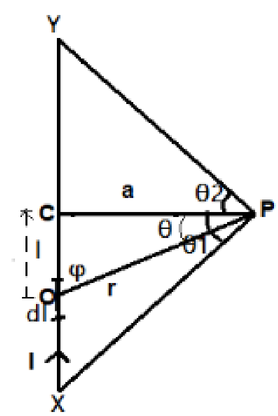
- Magnetic field at the axis of circular current carrying wire:

  $B = B_x\hat{i} = \dfrac{\mu_o IR^2}{2(x^2+R^2)^{3/2}}\hat{i}$

  And, at x=0, center of the loop $B_o = \dfrac{\mu_o I}{2R}\hat{i}$

- Magnetic field due to a current in straight conductor:

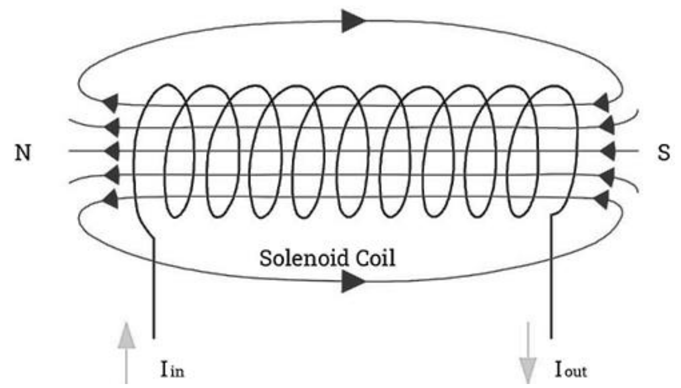
  $$B_0 = \frac{\mu_0 i}{4\pi r}\left(\sin(\theta_1) + \sin(\theta_2)\right)$$



- Magnetic field at perpendicular bisector of a straight current carrying conductor:
  $B_o = \frac{\mu_o}{4\pi}.\frac{i}{r}(2\sin\theta)$
- Magnetic field due to a semi-infinite wire: $B_o = \frac{\mu_o}{4\pi}.\frac{i}{r}$
- Magnetic field due to a wire at the axial position: B=0

Ampere's Circuital Law: $\oint B.dl = \mu_o I$

- Magnetic field due to an infinite long straight wire of radius ***a,*** current ***I*** and at a point ***r***:
  a. At r<a, $B = \frac{\mu_o Ir}{2\pi a^3}$
  b. r=a, $B = \frac{\mu_o I}{2\pi a}$
  c. r>a, $B = \frac{\mu_o I}{2\pi r}$
- Magnetic field due to solenoid:



$B = \mu_o nI = \frac{\mu_o NI}{L}, N =$ number of turns and L= length of rod

- Magnetic field due to Toroid: $B = \mu_o nI = \frac{\mu_o NI}{2\pi R}$
- Force between two parallel currents carrying wire I_1 and I_2 at distance d:

$$F_{21} = I_2 LB_2 = \frac{\mu_o I_1 I_2}{2\pi d} L \text{ and } F_{21} = -F_{12}$$

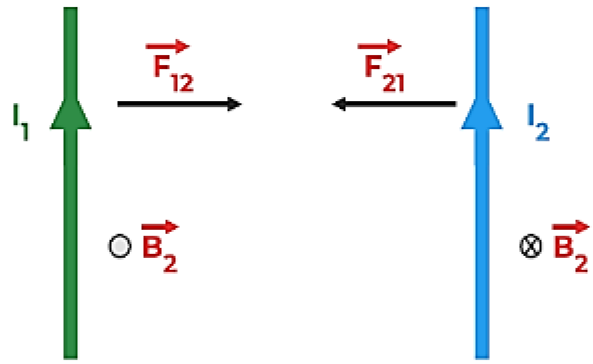
Torque on current loop, magnetic dipole:

- Torque on a rectangular current loop in uniform magnetic field:
  a. When plane of the loop is along with magnetic field: $\tau = IAB$, where A= area of rectangle and B= magnetic field.
  b. When plane of the loop is not along with magnetic field: $\tau = IAB\sin\theta$
- Magnetic moment of current loop is: $m = IA$
- Therefore, torque would be: $\tau = m \times B$
- In case of electrostatic then it has electric dipole of dipole moment p_e in electric field E: $\tau = p_e \times E$

Circular Current loop as a magnetic dipole: Magnetic field in terms of magnetic moment:

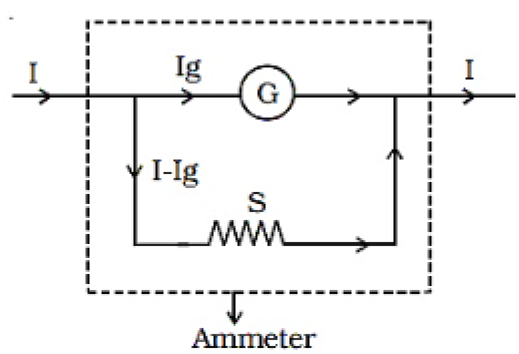
$B = \frac{\mu_o m}{2\pi x^3}$, where m=IA and x= distance from the dipole.

The magnetic dipole moment of a revolving electron: $\mu_l = \frac{e}{2m_e}(m_e vr) = \frac{e}{2m_e} l$ where $l = m_e vr$ and l= magnitude of the angular momentum, from Bohr's hypothesis $l = \frac{nh}{2\pi}$.



The moving coil galvanometer:

- Torque produced when current flows through the coil: $\tau = NIAB$
- Counter torque produced by spring that balances the magnetic torque: $k\phi = NIAB$ where k= torsional constant of the spring i.e., the restoring torque per unit turns and ϕ is deflection on the scale by a pointer attached to spring. Hence, $\phi = \frac{NAB}{k} I$
- **For measuring currents**, the galvanometer has to be connected in series. In this case we connect a low resistance called shunt in parallel with the galvanometer coil.



- **For measuring voltage**, the galvanometer also can be used as a voltmeter to measure the voltage throughout a given phase of the circuit. For this the voltmeter should be connected in parallel with that phase of the circuit.

$$\frac{\phi}{V}=\left(\frac{NAB}{k}\right)\frac{I}{V}=\left(\frac{NAB}{k}\right)\frac{1}{R}$$