Jupiter Academy

Subjects : Physics | PRACTICE PAPER 06

Total Marks : 180 | (Solutions)

Physics - Section A (MCQ)

1. The particles A and B of mass m each are separated by a distance r. Another particle C of mass M is placed at the midpoint of A and B. Find the work done in taking C to a point equidistant r from A and B without acceleration (G = Gravitational constant and only gravitational interaction between A, B and C is considered)

A) $\frac{GMm}{r}$
B) $\frac{2GMm}{r}$
C) $\frac{3GMm}{r}$
D) $\frac{4GMm}{r}$

Solution : (Correct Answer: B)

Since particle C is moved without any acceleration,

$\Rightarrow \Delta K.E = 0$

$\Rightarrow$ Work done by external agent $+W_{\text{gravitation}} = 0$

$\Rightarrow$ Work done by external agent $= -Wg$

$= -(-\Delta U)$

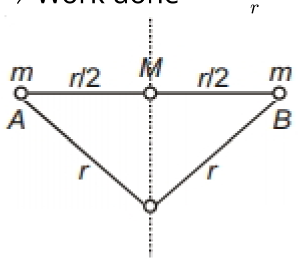
$= \Delta U$

$= U_f - U_{\text{in}}$

$U_f = -\frac{GMm}{r} - \frac{GMm}{r} = -\frac{2GMm}{r}$

$U_i = -\frac{GMm}{r/2} - \frac{GMm}{r/2} = -\frac{4GMm}{r}$

$\Rightarrow$ Work done $= \frac{2GMm}{r}$



2. If earth suddenly stop rotating, then the weight of an object of mass m at equator will [ω is angular speed of earth and R is its radius]

A) Decrease by $m\omega^2 R$
B) Increase by $m\omega^2 R$
C) Decrease by $m\omega R^2$
D) Increase by $m\omega R^2$

Solution : (Correct Answer: B)

(b)

At the equator,

Apparent weight, $w' = w - m\omega^2 R$

If Earth stops rotating, w' will be equal to ω.

Thus, the weight of an object of mass m at equator will increase by $m\omega^2 R$.

3. If a satellite of mass $400\ kg$ revolves around the earth in an orbit with speed $200\ m/s$ then its potential energy is .... MJ

A) -1.2
B) -8.0
C) -16
D) -2.4

Solution : (Correct Answer: C)

(c)

$PE = -2KE$

$= -2 \times 1/2 \times MV^2$

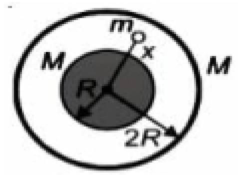
given,

mass is $400\, kg, V = 200\, m/s$

on substituting we get $PE = -400 \times 200 \times 200$

$= -16 \times 10^6\ J = -16$ megajoules $= -16\, MJ$

4. A uniform sphere of mass M and radius R is surrounded by a concentric spherical shell of same mass but radius $2R$. A point mass m is kept at a distance $x (> R)$ in the region bounded by spheres as shown in the figure. The net gravitational force on the particle is ......



A) $\frac{GMm}{x^2}$
B) $\frac{GMmx}{R^3}$
C) $\frac{G(M+m)}{x^2}$
D) Zero

Solution : (Correct Answer: A)

The gravitational force on the point mass m due to uniform sphere $= \frac{GMm}{x^2}$.

The gravitational force on the point mass due to the outer spherical shell is zero because gravitational force of attraction on a point mass due to various rejoins of the spherical shell cancels each other completely as their vector sum is zero.

5. The mass of the moon is $\frac{1}{81}$ of the earth but the gravitational pull is $\frac{1}{6}$ of the earth. It is due to the fact that

A) The radius of the moon is $\frac{81}{6}$ of the earth

B) The radius of the earth is $\frac{9}{\sqrt{6}}$ of the moon

C) Moon is the satellite of the earth

D) None of the above

Solution : (Correct Answer: B)

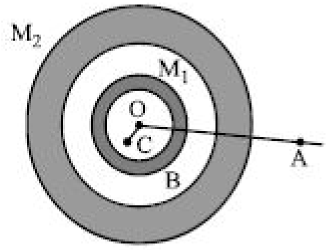
(b) Gravitational pull depends upon the acceleration due to gravity on that planet.

$M_m = \frac{1}{81}M_e, g_m = \frac{1}{6}g_e$

$g = \frac{GM}{R^2} ==> \frac{R_e}{R_m} = \left(\frac{M_e}{M_m} \times \frac{g_m}{g_e}\right)^{1/2} = \left(81 \times \frac{1}{6}\right)^{1/2}$

$\therefore R_e = \frac{9}{\sqrt{6}}R_m$

6. Two concentric shells of uniform density of mass M_1 and M_2 are situated as shown in the figure. The forces experienced by a particle of mass m when placed at positions A, B and C respectively are (given $OA = p$, $OB = q$ and $OC = r$)



A) zero, $G\frac{M_1 m}{q^2}$ and $G\frac{(M_1+M_2)m}{p^2}$

B) $G\frac{(M_1+M_2)m}{p^2}, G\frac{(M_1+M_2)m}{q^2}$ and $G\frac{M_1 m}{r^2}$

C) $G\frac{M_1 m}{q^2}, G\frac{(M_1+M_2)m}{p^2}$ and zero

D) $G\frac{(M_1+M_2)m}{p^2}, G\frac{M_1 m}{q^2}$ and zero

Solution : (Correct Answer: D)

zero, $G\frac{M_1 m}{q^2}$ and $G\frac{(M_1+m_2)m}{p^2}$

$G\frac{(m_1+M_2m)}{p^2}$ and $G\frac{(M_1+m_2)m}{q^2}$ and $G\frac{M_1 m}{r^2}$

$G\frac{M_1 m}{q^2}, G\frac{M_1 m}{q^2}$ and zero

$\frac{G(M_1+M_2)m}{p^2}, G\frac{M_1 m}{q^2}$ and zero

7. Match the column $-I$ with column $-II$ For a satellite in circular orbit,

Column $-I$	Column $-II$
(A) Kinetic energy	(p) $-\frac{GM_E m}{2r}$
(B) Potential energy	(q) $\sqrt{\frac{GM_E}{r}}$
(C) Total energy	(r) $-\frac{GM_E m}{r}$
(D) Orbital energy	(s) $\frac{GM_E m}{2r}$

(where M_E is the mass of the earth, m is mass of the satellite and r is the radius of the orbit)

A) $A-r, B-s, C-q, D-p$

B) $A-q, B-p, C-r, D-s$

C) $A-p, B-q, C-s, D-r$

D) $A-s, B-r, C-p, D-q$

Solution : (Correct Answer: D)

$A-s, B-r, C-p, D-q$

8. If the acceleration due to gravity at earth is $'g'$ and mass of earth is $80\ times$ that of moon and radius of earth is $4\ times$ that of moon, the value of acceleration due to gravity at the surface of moon will be

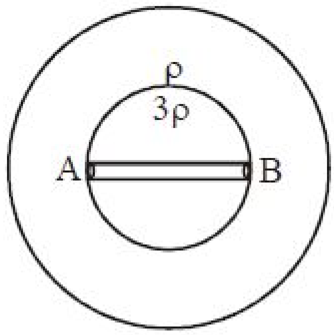
A) g B) $g/20$ C) $g/5$ D) $320g$

Solution : (Correct Answer: C)

$\mathrm{G_{moon}} = \frac{\mathrm{GMm}}{\mathrm{R_M^2}} = \frac{\mathrm{G(Me/80)}}{\mathrm{(Re/4)^2}}$

$= \frac{1}{5}\frac{\mathrm{GMe}}{\mathrm{Re^2}} = \frac{\mathrm{g}}{5}$

9. A planet of core density 3ρ and outer crust of density ρ has small tunnel in core. A small particle of mass m is released from end A then time required to reach end B



A) $\sqrt{\frac{\pi}{\rho G}}$ B) $\frac{1}{2}\sqrt{\frac{\pi}{\rho G}}$

C) $\pi\sqrt{\frac{1}{\rho G}}$ D) $2\pi\sqrt{\frac{1}{\rho G}}$

Solution : (Correct Answer: B)

at some distance from centre inside core

$\mathrm{F} = -\left(\frac{G\frac{4}{3}\pi r^3(3\rho)m}{r^2}\right)$

$\mathrm{ma} = -4\pi G\rho m r$

$\mathrm{a} = -4\pi G\rho r$

so $\omega = \sqrt{4\pi G\rho} = \frac{2\pi}{T}$

or $\mathrm{T} = 2\pi \cdot \sqrt{\frac{1}{4\pi G\rho}} = \sqrt{\frac{\pi}{G\rho}}$

now time for A to B $\frac{1}{2}\sqrt{\frac{\pi}{G\rho}}$

10. A missile is launched with a velocity less than the escape velocity. The sum of its kinetic and potential energy is

A) Positive

B) Negative

C) Zero

D) May be positive or negative depending upon its initial velocity

Solution : (Correct Answer: B)

(b) If missile launched with escape velocity then it will escape from the gravitational field and at infinity its total energy becomes zero.

But if the velocity of projection is less than escape velocity then sum of energies will be negative. This shows that attractive force is working on the satellite.

11. Two bodies of masses m_1 and m_2 are initially at rest at infinite distance apart. They are then allowed to move towards each other under mutual gravitational attraction. Their relative velocity of approach at a separation distance r between them is

A) $\left[2G\frac{(m_1-m_2)}{r}\right]^{1/2}$ B) $\left[\frac{2G}{r}(m_1+m_2)\right]^{1/2}$

C) $\left[\frac{r}{2G(m_1m_2)}\right]^{1/2}$ D) $\left[\frac{2G}{r}m_1m_2\right]^{1/2}$

Solution : (Correct Answer: B)

(b) Let velocities of these masses at r distance from each other be v_1 and v_2 respectively. By conservation of momentum $m_1v_1 - m_2v_2 = 0$

$\Rightarrow m_1v_1 = m_2v_2$... (i)

By conservation of energy change in $P.E.$ = change in $K.E.$

$\frac{Gm_1m_2}{r} = \frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2$

$\Rightarrow \frac{m_1^2v_1^2}{m_1} + \frac{m_2^2v_2^2}{m_2} = \frac{2\,Gm_1m_2}{r}$ ...(ii)

On solving equation (i) and (ii) $v_1 = \sqrt{\frac{2\,Gm_2^2}{r(m_1+m_2)}}$ and $v_2 = \sqrt{\frac{2\,Gm_1^2}{r(m_1+m_2)}}$

$\therefore\ v_{\text{app}} = |\,v_1\,| + |\,v_2\,| = \sqrt{\frac{2G}{r}(m_1+m_2)}$

12. If mass of earth decreases by 25% and its radius increases by 50%, then acceleration due to gravity at its surface decreases by nearly .... $\%$

A) 89 B) 67 C) 33 D) 11

Solution : (Correct Answer: B)

(b)

we know that

$g = \frac{GM}{R^2}$

The mass of the earth decrease by 25%,

remaining 75% of

$M = \frac{75}{100}\,M$

radius of earth increase by 50%,

new radius $= 150\%$ of $R = \frac{150}{100}R$

$g' = \frac{G\left(\frac{75}{100}M\right)}{\left(\frac{150R}{100}\right)^2}$

$g' = G^M\frac{M}{R^2}\left[\frac{75}{100}\times\frac{10000}{150\times150}\right]$

$g' = g\frac{5}{15}$

$g' = 0.33\,g$

The acceleration due to gravity decreases by

$= g - 0.33\,g$

$= 0.67\,g$

The acceleration due to gravity decreases by

$= 0.67 \times 100\%$

$= 67\,\%$

Hence the acceleration due to gravity at its surface decreases by nearly 67%.

13. If radius of earth contracted by 0.1%, its mass remaining same, then weight of the body at earth's surface will increase by ....$\%$

A) 0.1% B) 0.2%

C) 0.3% D) Remains same

Solution : (Correct Answer: B)

(b)

If radius is decreased by 0.1%

$r' = r - 0.001r = 0.999\,r$

$W' = \frac{GMm}{(0.999\,r)^2} = \frac{GMm}{(0.999)^2\,r^2}$

$= \frac{W}{0.999^2}$

$= 1.002\,W$

$\Delta W = W' - W = 0.002$

$0.002 \times 100 = 0.2\,\%$ increase.

14. A satellite moving round the earth in a circular orbit of radius r and speed v suddenly loses some of its energy. Then

A) r will increase and v will decrease

B) both r and v will decrease

C) both r and v will increase

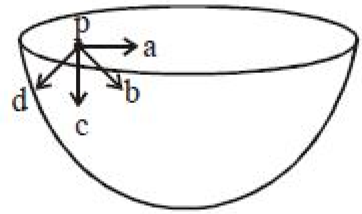
D) r will decrease and v will increase

Solution : (Correct Answer: D)

Given that, the particle lost some energy due to a collision. So, it can no longer continue in that orbit as the earth's gravitational force is more than centripetal force.

Due to this, the distance d decreases gradually and the particle moves towards earth at certain accelaration gaining speed. Thus distance d decreases and speed v increases.

15. Figure shows a hemispherical shell. The direction of gravitational field intensity at point p will be along



A) a B) b C) c D) d

Solution : (Correct Answer: C)

16. When a satellite in a circular orbit around the earth enters the atmospheric region, it encounters small air resistance to its motion. Then

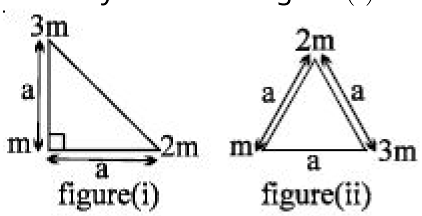
A) its kinetic energy increases
B) its kinetic energy decreases
C) its angular momentum about the earth decreases
D) (A) and (C) both

Solution : (Correct Answer: D)

$KE = \frac{GMm}{2r} \uparrow$

Due to air resistance angular mometum will decreases $T = 2\pi\sqrt{\frac{r^3}{GM}}$, so option (A) and (C) are correct.

17. Consider two configurations of a system of three particles of masses $m, 2m$ and $3m$. The work done by external agent in changing the configuration of the system from figure (i) to figure (ii) is



A) zero
B) $-\frac{6Gm^2}{a}\left(1+\frac{1}{\sqrt{2}}\right)$
C) $-\frac{6Gm^2}{a}\left(1-\frac{1}{\sqrt{2}}\right)$
D) $-\frac{6Gm^2}{a}\left(2-\frac{1}{\sqrt{2}}\right)$

Solution : (Correct Answer: C)

$U_1 = 0 - \frac{Gm2m}{a} - \frac{Gm3m}{a} - \frac{G3m2m}{a}$

$= -\frac{5Gm^2}{a} - \frac{3\sqrt{2}m^2}{a}$

$= -\frac{Gm^2}{a}(5+3\sqrt{2})$

$U_2 = 0 - \frac{Gm3m}{a} - \frac{Gm2m}{a} - \frac{G3m2m}{a}$

$= -\frac{11Gm^2}{a}$

$\Delta W = \Delta U = \frac{Gm^2}{a}(11-5-3\sqrt{2})$

$= \frac{6Gm^2}{a}\left(1-\frac{1}{\sqrt{2}}\right)$

18. The mass and diameter of a planet are twice those of earth. What will be the period of oscillation of a pendulum on this planet if it is a seconds pendulum on earth ?

A) $\sqrt{2}$ second B) $2\sqrt{2}$ seconds
C) $\frac{1}{\sqrt{2}}$ second D) $\frac{1}{2\sqrt{2}}$ second

Solution : (Correct Answer: B)

$g = \frac{GM}{R^2}$ and $g' = \frac{G \cdot 2M}{4R^2}$

$= \frac{1}{2}g$

$T \propto \frac{1}{\sqrt{g}} \Rightarrow \frac{T_2}{T_1} = \sqrt{\frac{g_1}{g_2}}$

$T = \sqrt{\frac{g}{g/2}} = \sqrt{2}$

$\therefore T_2 = \sqrt{2}T_1 = 2\sqrt{2}\mathrm{s}$

19. A particle is projected from the mid - point of the line joining two fixed particles each of mass $'m'$. If the separation between fixed particles is $'l'$, the minimum velocity of projection of the particle so as to escape is equal to

A) $\sqrt{\frac{Gm}{l}}$ B) $\sqrt{\frac{Gm}{2l}}$
C) $\sqrt{\frac{2Gm}{l}}$ D) $2\sqrt{\frac{2Gm}{l}}$

Solution : (Correct Answer: D)

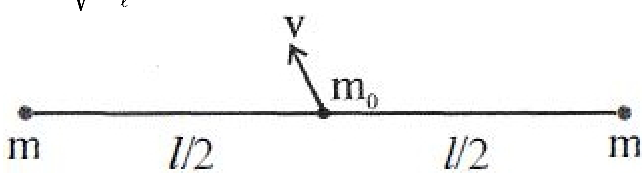
Gravitational potential energy

$= \frac{-4Gmm_0}{\ell}$

to escape, $\Delta\mathrm{KE} + \Delta\mathrm{PE} = 0$

$= \left(0 - \frac{1}{2}m_0v^2\right) + \left\{0 - \left(-\frac{4Gmm_0}{\ell}\right)\right\} = 0$

$v = 2\sqrt{\frac{2Gm}{\ell}}$



20. If the radius of the earth be increased by a factor of 5, by what factor its density be changed to keep the value of g the same ?

A) $1/25$ B) $1/5$ C) $1/\sqrt{5}$ D) 5

Solution : (Correct Answer: B)

The acceleration due to gravity on surface of earth is

$g = \frac{GM}{R^2}$

$g = \frac{G\rho V}{R^2}$

where, G is the gravitational constant, M is mass of earth, R is radius of earth, ρ is the density and V is the volume of earth.

$\therefore g = \frac{G\rho\frac{4}{3}\pi R^2}{R^2}$

$\therefore g = \frac{4}{3}G\rho\pi R$

$\Rightarrow g \propto \rho R$

since, all other quantities are constant.

Hence, if the radius of the earth be increased by a factor of 5, density will decrease by $\frac{1}{5}$ to keep the value of g the same.

21. What does not change in the field of central force

A) Potential energy B) Kinetic energy
C) Linear momentum D) Angular momentum

Solution : (Correct Answer: D)

(d) For central force, torque is zero.

$\tau = \frac{dL}{dt} = 0$

$L =$ constant

i.e. Angular momentum is constant.

22. A satellite of mass m is circulating around the earth with constant angular velocity. If radius of the orbit is R_0 and mass of the earth M, the angular momentum about the centre of the earth is

A) $m\sqrt{GMR_0}$ B) $M\sqrt{GmR_0}$
C) $m\sqrt{\frac{GM}{R_0}}$ D) $M\sqrt{\frac{GM}{R_0}}$

Solution : (Correct Answer: A)

(a) Angular momentum = Mass $\times$ Orbital velocity $\times$ Radius

$= m \times \left(\sqrt{\frac{GM}{R_0}}\right) \times R_0 = m\sqrt{GMR_0}$

23. The time period of a satellite of earth is $5\,hours$. If the separation between the centre of earth and the satellite is increased to $4\,times$ the previous value, the new time period will become .... h

A) 10 B) 80 C) 40 D) 20

Solution : (Correct Answer: C)

$\mathrm{T} \propto \mathrm{r}^{\frac{3}{2}}$

$\frac{\mathrm{T}^1}{\mathrm{T}} = (4)^{\frac{3}{2}}$

$\mathrm{T}^1 = 5 \times 8 = 40$

24. A hollow spherical shell is compressed to half its radius. The gravitational potential at the centre

A) increases
B) decreases
C) remains same
D) during the compression increases then returns at the previous value.

Solution : (Correct Answer: B)

$V = -\frac{GM}{r}$

Due to the negative sign, when the radius is halved the potential decreases.

25. A communications Earth satellite

A) goes round the earth from west to east
B) can be in the equatorial plane only
C) can be vertically above any place on the earth
D) (A) and (B) both

Solution : (Correct Answer: D)

26. If V, R and g denote respectively the escape velocity from the surface of the earth radius of the earth, and acceleration due to gravity, then the correct equation is

A) $V = \sqrt{gR}$ B) $V = \sqrt{\frac{4}{3}gR^3}$
C) $V = R\sqrt{g}$ D) $V = \sqrt{2gR}$

Solution : (Correct Answer: D)

To escape the surface of the earth, the kinetic energy of an object has to be equal to the work done against the gravity going from the surface to infinity.

So, $\frac{1}{2}mv_e^2 = \frac{GMm}{R}\left[\begin{array}{ll} M = & \text{mass of earth} \\ R = & \text{Radius of earth} \end{array}\right]$

$\therefore v_e = \sqrt{\frac{2GM}{R}} = \sqrt{2gR} \quad (\because g = \frac{GM}{R^2})$

This will be the required equation.

27. Orbit of a planet around a star is

A) A circle B) An ellipse
C) A parabola D) A straight line

Solution : (Correct Answer: B)

(b)

28. If $g \propto \frac{1}{R^3}$ (instead of $\frac{1}{R^2}$), then the relation between time period of a satellite near earth's surface and radius R will be

A) $T^2 \propto R^3$
B) $T \propto R^2$
C) $T^2 \propto R$
D) $T \propto R$

Solution : (Correct Answer: B)

(b)Gravitational force provides the required centripetal force

$m\omega^2 R = \frac{GMm}{R^3}$

$\Rightarrow \frac{4\pi^2}{T^2} = \frac{GM}{R^4}$

$\Rightarrow T \propto R^2$

29. A person sitting in a chair in a satellite feels weightless because

A) The earth does not attract the objects in a satellite

B) The normal force by the chair on the person balances the earth's attraction

C) The normal force is zero

D) The person in satellite is not accelerated

Solution : (Correct Answer: C)

(c) Because the both the chair and the man are in a state of free fall and have similar forces of gravity acting on them. This equates to relative acceleration being zero between the man and chair leading to a zero value of normal force.

30. A satellite revolves around the earth in an elliptical orbit. Its speed

A) Is the same at all points in the orbit

B) Is greatest when it is closest to the earth

C) Is greatest when it is farthest from the earth

D) Goes on increasing or decreasing continuously depending upon the mass of the satellite

Solution : (Correct Answer: B)

(b) Speed of satellite is greatest when it is closest to earth as near earth surface r

$\approx \mathrm{R}, \mathrm{V}_0 = \sqrt{(\mathrm{GM}/\mathrm{r})} = 7.9\mathrm{kms}^{-1}$

This is maximum speed of satellite

31. A satellite which is geostationary in a particular orbit is taken to another orbit. Its distance from the centre of earth in new orbit is 2 times that of the earlier orbit. The time period in the second orbit is

A) 4.8 hours
B) $48\sqrt{2}$ hours
C) 24 hours
D) $24\sqrt{2}$ hours

Solution : (Correct Answer: B)

(b) $T \propto r^{3/2}$. If r becomes double then time period will becomes $(2)^{3/2}$ times.

So new time period will be $24 \times 2\sqrt{2}\ hr$ i.e. $T = 48\sqrt{2}$

32. Select the correct statement from the following

A) The orbital velocity of a satellite increases with the radius of the orbit

B) Escape velocity of a particle from the surface of the earth depends on the speed with which it is fired

C) The time period of a satellite does not depend on the radius of the orbit

D) The orbital velocity is inversely proportional to the square root of the radius of the orbit

Solution : (Correct Answer: D)

(d)$v_0 = \sqrt{\frac{GM}{r}}$

33. There are two planets. The ratio of radius of the two planets is K but ratio of acceleration due to gravity of both planets is g. What will be the ratio of their escape velocity

A) $(Kg)^{1/2}$
B) $(Kg)^{-1/2}$
C) $(Kg)^2$
D) $(Kg)^{-2}$

Solution : (Correct Answer: A)

(a)$v = \sqrt{2gR}$

$\frac{v_1}{v_2} = \sqrt{\frac{g_1}{g_2} \times \frac{R_1}{R_2}} = \sqrt{g \times K}$= $(Kg)^{1/2}$

34. At what distance from the centre of the earth, the value of acceleration due to gravity g will be half that on the surface ($R =$ radius of earth)

A) $2\,R$
B) R
C) $0.414\,R$
D) $1.414\,R$

Solution : (Correct Answer: D)

(d) $g' = g\left(\frac{R}{R+h}\right)^2 \Rightarrow \frac{1}{\sqrt{2}} = \frac{R}{R+h}$

$\Rightarrow R + h = \sqrt{2}\,R \Rightarrow h = (\sqrt{2} - 1)R = 0.414\,R$

$R + h = 1.414\,R$

35. In a gravitational field, at a point where the gravitational potential is zero

A) The gravitational field is necessarily zero

B) The gravitational field is not necessarily zero

C) Nothing can be said definitely about the gravitational field

D) None of these

Solution : (Correct Answer: A)

(a) $I = \frac{-dV}{dx}$

If $V = 0$ then gravitational field is necessarily zero.

36. A body weight $500\ N$ on the surface of the earth. How much would it weigh half way below the surface of the earth .... N

A) 125
B) 250
C) 500
D) 1000

Solution : (Correct Answer: B)

(b) Weight on surface of earth, $mg = 500\,N$ and weight below the surface of earth at $d = \frac{R}{2}$

$mg' = mg\left(1 - \frac{d}{R}\right) = mg\left(1 - \frac{1}{2}\right) = \frac{mg}{2} = 250\,N$

37. The angular velocity of the earth with which it has to rotate so that acceleration due to gravity on 60^o latitude becomes zero is (Radius of earth $= 6400\,km$. At the poles $g = 10\,ms^{-2}$)

A) $2.5 \times 10^{-3}\,rad/s$
B) $5.0 \times 10^{-1}\,rad/s$
C) $10 \times 10^{1}\,rad/s$
D) $7.8 \times 10^{-2}\,rad/s$

Solution : (Correct Answer: A)

(a) $g' = g - \omega^2 R\cos^2\lambda \Rightarrow 0 = g - \omega^2 R\cos^2 60^o$

$0 = g - \frac{\omega^2 R}{4} \Rightarrow \omega = 2\sqrt{\frac{g}{R}} = \frac{1}{400}\,\frac{rad}{\text{sec}}$

38. The diameters of two planets are in the ratio $4:1$ and their mean densities in the ratio $1:2$. The acceleration due to gravity on the planets will be in ratio

A) $1:2$ B) $2:3$ C) $2:1$ D) $4:1$

Solution : (Correct Answer: C)

(c) $g = \frac{4}{3}G\pi R\rho \Rightarrow \frac{g_1}{g_2} = \frac{\rho_1 R_1}{\rho_2 R_2} = \frac{1}{2} \times \frac{4}{1} = \frac{2}{1}$

39. Mass of moon is $7.34 \times 10^{22}\,kg$. If the acceleration due to gravity on the moon is $1.4\,m/s^2$, the radius of the moon is $(G = 6.667 \times 10^{-11}\,Nm^2/kg^2)$

A) $0.56 \times 10^4\,m$ B) $1.87 \times 10^6\,m$
C) $1.92 \times 10^6\,m$ D) $1.01 \times 10^8\,m$

Solution : (Correct Answer: B)

(b) $g = \frac{GM}{R^2} \Rightarrow R = \sqrt{\frac{GM}{g}}$

Substituting the above values we get $R = 1.87 \times 10^6 m.$

40. Where will it be profitable to purchase 1 kilogram sugar

A) At poles
B) At equator
C) At 45° latitude
D) At 40° latitude

Solution : (Correct Answer: B)

(b)Weight is least at the equator.

41. R is the radius of the earth and ω is its angular velocity and g_p is the value of g at the poles. The effective value of g at the latitude $\lambda = 60^\circ$ will be equal to

A) $g_p - \frac{1}{4}R\omega^2$
B) $g_p - \frac{3}{4}R\omega^2$
C) $g_p - R\omega^2$
D) $g_p + \frac{1}{4}R\omega^2$

Solution : (Correct Answer: A)

(a) $g = g_p - R\omega^2\cos^2\lambda = g_p - \omega^2 R\cos^2 60^\circ = g_p - \frac{1}{4}R\omega^2$

42. A planet is revolving around the sun in a circular orbit with a radius r. The time period is T. If the force between the planet and star is proportional to $r^{-3/2}$, then the square of time period is proportional to

A) $r^{3/2}$ B) r^2
C) r D) $r^{5/2}$

Solution : (Correct Answer: D)

As, force $= \frac{GMm}{r^{3/2}} = m\omega^2 r$

$= \frac{GMm}{r^{3/2}} = \frac{4\pi^2 mr}{T^2}\left[\because T = \frac{2\pi}{\omega}\right]$

$\Rightarrow T^2 = \left(\frac{\Delta\pi^2}{GM}\right) \cdot r^{5/2}$

$\Rightarrow T^2 \propto r^{5/2}$

43. $Assertion$: The earth without atmosphere would be inhospitably cold.

$Reason$: All heat would escape in the absence of atmosphere.

A) If both Assertion and Reason are correct and the Reason is a correct explanation of the Assertion.
B) If both Assertion and Reason are correct but Reason is not a correct explanation of the Assertion.
C) If the Assertion is correct but Reason is incorrect.
D) If both the Assertion and Reason are incorrect.

Solution : (Correct Answer: A)

In the absence of atmosphere, all the heat will escape from earth's surface which will make earth in hospitably cold.

44. A planet moving along an elliptical orbit is closest to the sun at a distance r_1 and farthest away at a distance of r_2. If v_1 and v_2 are the linear velocities at these points respectively, then the ratio $\frac{v_1}{v_2}$ is

A) $\frac{r_2}{r_1}$ B) $\left(\frac{r_2}{r_1}\right)^2$
C) $\frac{r_1}{r_2}$ D) $\left(\frac{r_1}{r_2}\right)^2$

Solution : (Correct Answer: A)

According to the Law of conservation of angular momentum

$mv_1r_1 = mv_2r_2$

$v_1r_1 = v_2r_2$

$\frac{v_1}{v_2} = \left(\frac{r_2}{r_1}\right)$

45. The velocity with which a projectile must be fired so that it escapes earth's gravitation does not depend on

A) Mass of the earth
B) Mass of the projectile
C) Radius of the projectile's orbit
D) Gravitational constant

Solution : (Correct Answer: B)

(b) At a certaing velocity projection, the body will go out of the gravitational field of the earth and will never return to the earth. Thic initial velocity is called escape velocity. The kinetic energy given to the body should be equal to potential energy for body to escape. i.e., potentiial energy $=$kinetic energy

$+\frac{GM_e m}{R} = \frac{1}{2}mv_e^2$

Where m is mass of projectile, M_e is mass of earth, G is gravitational constant, R is radius.

$$\therefore v_e = \sqrt{\frac{2GM_e}{R_e}}$$

The above formula shows that escape velocity is independent of the mass of the projectile.