Jupiter Academy

Subjects : Physics

PRACTICE PAPER 05

Total Marks : 180

(Solutions)

Physics - Section A (MCQ)

1. Two rings of same mass and radius R are placed with their planes perpendicular to each other and centres at a common point. The radius of gyration of the system about an axis passing through the centre and perpendicular to the plane of one ring is ....

A) $2R$
B) $\frac{R}{\sqrt{2}}$
C) $\sqrt{\frac{3}{2}}R$
D) $\frac{\sqrt{3}R}{2}$

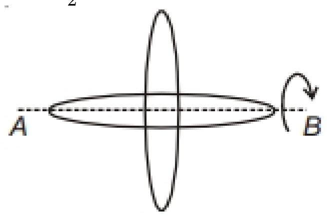
Solution : (Correct Answer: D)

$I_{AB} = mr^2 + \frac{mr^2}{2}$

$= \frac{3mr^2}{2}$

$\frac{3mr^2}{2} = 2mk^2$

$k = \frac{\sqrt{3}}{2}R$



2. Consider the following statements
Assertion (A) : The moment of inertia of a rigid body reduces to its minimum value as compared to any other parallel axis when the axis of rotation passes through its centre of mass.
Reason (R): The weight of a rigid body always acts through its centre of mass in uniform gravitational field. Of these statements:

A) both A and R are true and R is the correct explanation of A
B) both A and R are true but R is not a correct explanation of A
C) A is true but R is false
D) A is false but R is true

Solution : (Correct Answer: B)

3. The moment of inertia of semicircular ring about its centre is

A) MR^2
B) $\frac{MR^2}{2}$
C) $\frac{MR^2}{4}$
D) None of these

Solution : (Correct Answer: A)

$\partial m = \frac{m}{\pi R} \cdot R\partial\theta$

$\partial m = \frac{m}{\pi}\partial\theta$

$\partial I = \partial m \cdot R^2$

$I = \frac{m}{\pi}R^2 \int_0^{\pi} \partial\theta$

$= \frac{m}{\pi}R^2(\theta)_0^{\pi}$

$I = mR^2$

4. Two men are carrying a uniform bar of length L, on their shoulders. The bar is held horizontally such that younger man gets $(1/4)^{th}$ load. Suppose the younger man is at the end of the bar, what is the distance of the other man from the end

A) $L/3$
B) $L/2$
C) $2L/3$
D) $3L/4$

Solution : (Correct Answer: C)

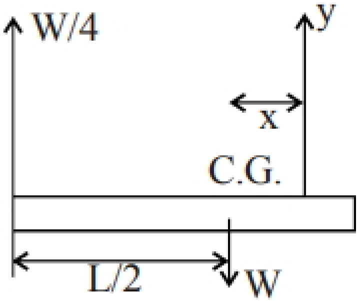
For the balance of forces, $y + w/4 = w$, therefore $y = 3w/4$

Further for a balance of torque about CG we get,

$\mathrm{W/4 \times L/2 = 3W/4 \times x}$ therefore $\mathrm{x = 1/6}$

Thus distance from the left end of the rod

$= \mathrm{L/2 + L/6 = 4L/6 = 2L/3}$



5. The coordinates of the positions of particles of mass $7, 4 \text{and} 10\, gm$ are $(1, 5, -3)$, $(2, 5, 7)$ and $(3, 3, -1)\, cm$ respectively. The position of the centre of mass of the system would be

A) $\left(-\frac{15}{7}, \frac{85}{17}, \frac{1}{7}\right) cm$
B) $\left(\frac{15}{7}, -\frac{85}{17}, \frac{1}{7}\right) cm$
C) $\left(\frac{15}{7}, \frac{85}{21}, -\frac{1}{7}\right) cm$
D) $\left(\frac{15}{7}, \frac{85}{21}, \frac{7}{3}\right) cm$

Solution : (Correct Answer: C)

$m_1 = 7\ gm$, $m_2 = 4\ gm$, $m_3 = 10\ gm$ and $\overrightarrow{r_1} = (\hat{i} + 5\hat{j} - 3\hat{k})$, $r_2 = (2i + 5j + 7k)$, $r_3 = (3\hat{i} + 3\hat{j} - \hat{k})$

Position vector of center mass $\overrightarrow{r}$

$= \frac{7(\hat{i}+5\hat{j}-3\hat{k})+4(2\hat{i}+5\hat{j}+7\hat{k})+10(3\hat{i}+3\hat{j}-\hat{k})}{7+4+10} = \frac{(45\hat{i}+85\hat{j}-3\hat{k})}{21}$

$\overrightarrow{r} = \frac{15}{7}\hat{i} + \frac{85}{21}\hat{j} - \frac{1}{7}\hat{k}.$

So coordinates of centre of mass $\left[\frac{15}{7}, \frac{85}{21}, \frac{-1}{7}\right]$

6. The moment of inertia of a thin circular lamina of mass $1\ kg$ and diameter $0.2\ metre$ rotating about one of its diameter is

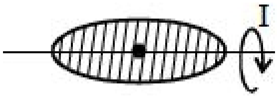
A) $5 \times 10^{-3}\ kg - m^2$
B) $2.5 \times 10^{-3}\ kg - m^2$
C) $4 \times 10^{-2}\ kg - m^2$
D) $0.2\ kg - m^2$

Solution : (Correct Answer: B)

$I = \frac{MR^2}{4}$

$I=\frac{(1)(0.1)^2}{4}$

$=2.5\times10^{-3}\mathrm{kg}-\mathrm{m}^2$



7. If $I=50\,kg-m^2$, then .... $N-m$ torque will be applied to stop it in $10\ sec$. Its initial angular speed is $20\,rad/sec$

A) 100 B) 150 C) 200 D) 250

Solution : (Correct Answer: A)

$\tau=I\alpha=\frac{I\Delta W}{\Delta t}=\frac{50(0-20)}{10}$

$\tau=-100\mathrm{N}-\mathrm{m}$ means

$100\mathrm{N}-\mathrm{m}$ in opposite to angular speed.

8. Two circular discs $\sqrt{2gh}$ and B are of equal masses and thickness but made of metals with densities d_A and d_B $(d_A>d_B)$. If their moments of inertia about an axis passing through centres and normal to the circular faces be I_A and I_B, then

A) $I_A=I_B$
B) $I_A>I_B$
C) $I_A<I_B$
D) $I_A>=<I_B$

Solution : (Correct Answer: C)

Moment of inertia of circular disc about an axis passing through centre and normal to the circular face

$I=\frac{1}{2}MR^2=\frac{1}{2}M\left(\frac{M}{\pi\,t\rho}\right)$ [As $M=V\rho=\pi\,R^2t\rho$, $R^2=\frac{M}{\pi\,t\,\rho}$]

$I=\frac{M^2}{2\pi\,t\rho}$ or $I\propto\frac{1}{\rho}$ If mass and thickness are constant

So, in the problem $\frac{I_A}{I_B}=\frac{d_B}{d_A}$ $\therefore\ I_A<I_B$ [As $d_A>d_B$)]

9. The velocity of the centre of mass of a rigid rod with respect to an observer O is $\vec{v}=\left(2\hat{i}+3\hat{j}\right)ms^{-1}$. The rod has an angular velocity about its centre of mass given by $\vec{\omega}=\left(3\hat{j}+4\hat{k}\right)s^{-1}$. Let A be a point on the rod with position vector $2\left(\hat{i}+\hat{k}\right)m$ with respect to the centre of mass. The velocity of the point A with respect to O is

A) $6\hat{i}+11\hat{j}+6\hat{k}$ B) $8\hat{i}+11\hat{j}-6\hat{k}$
C) $6\hat{i}-11\hat{j}+12\hat{k}$ D) $8\hat{i}+11\hat{j}-8\hat{k}$

Solution : (Correct Answer: B)

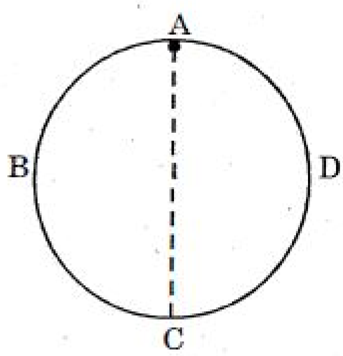
$\overrightarrow{\mathrm{v}}_{\mathrm{A/cm}}=\overrightarrow{\mathrm{v}}_{\mathrm{A}}-\overrightarrow{\mathrm{v}}_{\mathrm{cm}}\Rightarrow\overrightarrow{\mathrm{v}}_{\mathrm{A}}=\overrightarrow{\mathrm{v}}_{\mathrm{cm}}+\overrightarrow{\mathrm{v}}_{\mathrm{A/cm}}$

But $\overrightarrow{\mathrm{v}}_{\mathrm{A/cm}}=\vec{\omega}\times\overrightarrow{\mathrm{r}}=\begin{vmatrix}\hat{\mathrm{i}}&\hat{\mathrm{j}}&\hat{\mathrm{k}}\\0&3&4\\2&0&2\end{vmatrix}=6\hat{\mathrm{i}}+8\hat{\mathrm{j}}-6\hat{\mathrm{k}}$

Therefore

$\overrightarrow{\mathrm{v}}_{\mathrm{A}}=(2\hat{\mathrm{i}}+3\hat{\mathrm{j}})+(6\hat{\mathrm{i}}+8\hat{\mathrm{j}}-6\hat{\mathrm{k}})=8\hat{\mathrm{i}}+11\hat{\mathrm{j}}-6\hat{\mathrm{k}}$

10. A ring is formed by joining two uniform semi circular rings ABC and ADC. Mass of ABC is thrice of that of ADC. If the ring is hinged to a fixed support ,at A, it can rotate freely in a vertical plane. Find the value of $tan\,\theta$, where θ is the angle made. by the line AC with the vertical in equilibrium

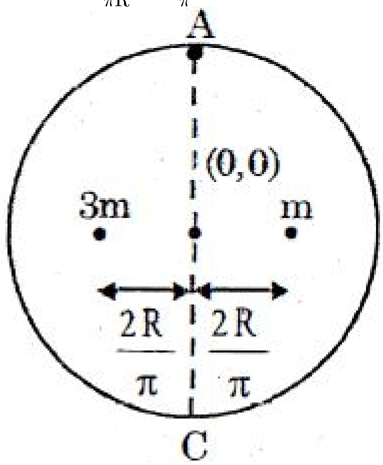


A) $\frac{9\pi}{4}$ B) $\frac{1}{\pi}$ C) $\frac{2}{3\pi}$ D) π

Solution : (Correct Answer: B)

$\mathrm{X_{com}}=\frac{-\mathrm{R}}{\pi}$

$\tan\theta=\frac{\mathrm{R}}{\pi\mathrm{R}}=\frac{1}{\pi}$



11. A rod of mass m and length l is placed on smooth floor pivoted at its centre. A gun is mounted at one end, of mass m without a shot. It fires a shot of mass $\frac{m}{10}$ with velocity v relative to gun. The angular velocity of the rod will be

A) $\frac{6}{23}\frac{v}{l}$ B) $\frac{6}{43}\frac{v}{l}$
C) $\frac{3}{23}\frac{v}{l}$ D) $\frac{7}{23}\frac{v}{l}$

Solution : (Correct Answer: B)

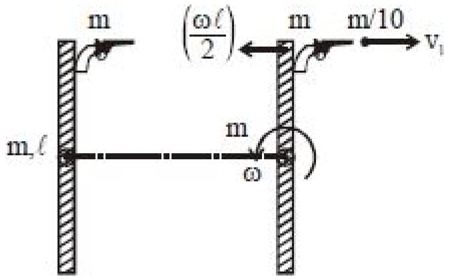
$v_1+\omega\frac{\ell}{2}=v(\text{ given })$

angular momentum conservation

$\frac{\mathrm{m}}{10}\mathrm{v}_1\frac{\ell}{2}=\left[\frac{\mathrm{m}\ell^2}{12}+\mathrm{m}\left(\frac{\ell}{2}\right)^2\right]\omega$

$\frac{1}{10}\left(v-\frac{\omega \ell}{2}\right)\frac{\ell}{2}=\left(\frac{\ell^2}{12}+\frac{\ell^2}{4}\right)\omega$

$\Rightarrow \omega=\frac{6v}{43\ell}$



12. A body is rolling down an inclined plane. If kinetic energy of rotation is 40% of translational kinetic energy, then the body is a

A) Ring B) Cylinder
C) Hollow ball D) Solid ball

Solution : (Correct Answer: D)

Rotational kinetic energy is

$\mathrm{K_R}=\frac{1}{2}\mathrm{I}\omega^2=\frac{1}{2}\mathrm{Mk}^2\left(\frac{\mathrm{v}}{\mathrm{R}}\right)^2\left(\because \mathrm{I}=\mathrm{Mk}^2 \text{ and } \mathrm{v}=\mathrm{R}\omega\right)$

$=\frac{1}{2}\mathrm{Mv}^2\left(\frac{\mathrm{k}^2}{\mathrm{R}^2}\right)$

Translational kinetic energy is

$\mathrm{K_T}=\frac{1}{2}\mathrm{Mv}^2$

As per questions, $\mathrm{K_R}=40\%\mathrm{K_T}$

$\therefore \frac{1}{2}\mathrm{Mv}^2\left(\frac{\mathrm{k}^2}{\mathrm{R}^2}\right)=40\%\frac{1}{2}\mathrm{Mv}^2$ or $\frac{\mathrm{k}^2}{\mathrm{R}^2}=\frac{40}{100}=\frac{2}{5}$

For solid sphere, $\frac{\mathrm{k}^2}{\mathrm{R}^2}=\frac{2}{5}$

Hence, the body is solid ball.

13. The angular momentum of a flywheel having a moment of inertia of $0.4\,kg\,m^2$ decreases from 30 to $20\ kg\ m^2/s$ in a period of 2 second . The average torque acting on the flywheel during this period is .... $N.m$

A) 10 B) 2.5 C) 5 D) 1.5

Solution : (Correct Answer: C)

$\bar{\tau}=\frac{\Delta \mathrm{L}}{\Delta \mathrm{t}}$

$\bar{\tau}=\frac{10}{2}=5\mathrm{Nm}$

14. For a system to be in equilibrium, the torques acting on it must balance. This is true only if the torques are taken about

A) The centre of the system
B) The centre of mass of the system
C) Any point on the system
D) Any point on the system or outside it

Solution : (Correct Answer: D)

For a system to be in equilibrium torques on the system must be equal to zero This is true only if the torques are taken about centre of mass as the whole

15. Two blocks of masses $5\,kg$ and $2\,kg$ are connected by a spring of negilible mass and placed on a frictionless horizontal surface. An impulse gives a velocity of $7\ m/s$ to the heavier block in the direction of the lighter block. The velocity of the centre of mass is .... m/s

A) 30 B) 20 C) 10 D) 5

Solution : (Correct Answer: D)

(d)

$v_{cm}=\frac{m_1v_1+m_2v_2}{m_1+m_2}$

$=\frac{5(7)+2(0)}{7}=5\,m/s$

16. A solid cylinder is rolling down on an inclined plane of angle θ. The coefficient of static friction between the plane and the cylinder is μ_s. The condition for the cylinder not to slip is

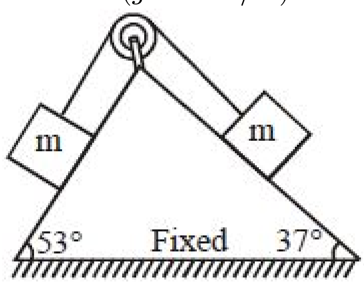
A) $\tan\theta \geq 3\mu_s$
B) $\tan\theta > 3\mu_s$
C) $\tan\theta \leq 3\mu_s$
D) $\tan\theta < 3\mu_s$

Solution : (Correct Answer: C)

$\mu_\mathrm{s}\geq \frac{\tan\theta}{1+\mathrm{R}^2/\mathrm{K}^2}\Rightarrow \mu_\mathrm{s}\geq\frac{\tan\theta}{1+2}\rightarrow \tan\theta\leq 3\mu_\mathrm{s}$

$\frac{\mathrm{K}^2}{\mathrm{R}^2}=\frac{1}{2}(\text{ solid cylinder })$

17. Two blocks which are connected to each other by means of a massless string are placed on two inclined planes as shown in figure. After releasing from rest, the magnitude of acceleration of the centre of mass of both the blocks is $(g=10\,m/s^2)$



A) $1\,m/s^2$ B) $\frac{1}{\sqrt{2}}\,m/s^2$
C) $\sqrt{2}\,m/s^2$ D) Zero

Solution : (Correct Answer: B)

$a_1=a_2=\frac{mg\sin 53^\circ-\mathrm{mg}\sin 37^\circ}{2m}=\frac{g}{10}=1\mathrm{m/s}^2$

$a_{cm}=\frac{m\sqrt{a_1^2+a_2^2}}{2m}=\frac{\sqrt{2}}{2}=\frac{1}{\sqrt{2}}\mathrm{m/s}^2$

18. Consider a particle of mass m having linear momentum $\vec{p}$ at position $\vec{r}$ relative to the origin O . Let $\vec{L}$ be the angular momentum of the particle with respect the origin. Which of the following equations correctly relate $(s)\,\vec{r}$, $\vec{p}$ and $\vec{L}$?

A) $\frac{d\vec{L}}{dt}+\vec{r}\times\frac{\overrightarrow{dp}}{dt}=0$
B) $\frac{d\vec{L}}{dt}+\frac{\overrightarrow{dr}}{dt}\times\vec{p}=0$
C) $\frac{d\vec{L}}{dt}-\frac{\overrightarrow{dr}}{dt}\times\vec{p}=0$
D) $\frac{d\vec{L}}{dt}-\vec{r}\times\frac{\overrightarrow{dp}}{dt}=0$

Solution : (Correct Answer: D)

As $\overrightarrow{L}=\overrightarrow{r}\times\overrightarrow{p}$

Differentiate both sides with respect to time, we get

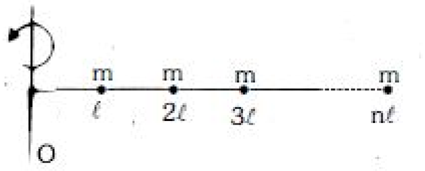
$$\frac{d\overrightarrow{L}}{dt}=\frac{d}{dt}(\overrightarrow{r}\times\overrightarrow{p})$$

$$=\frac{d\vec{r}}{dt}\times\vec{p}+\vec{r}\frac{\overrightarrow{dp}}{dt}$$

$$=\overrightarrow{r}\times\frac{\overrightarrow{dt}}{dt}\left(\because\frac{\overrightarrow{dr}}{dt}\times\overrightarrow{p}=0\right)$$

$$\frac{d\vec{L}}{dt}-\vec{r}\times\frac{d\vec{p}}{dt}=0$$

19. Find moment of inertia about axis OA



A) $\frac{ml^2}{6}$
B) $\frac{ml^2}{6}n\,(n+1)\,(2n+1)$
C) zero
D) $ml^2\frac{n(n+1)}{2}$

Solution : (Correct Answer: B)

$$\Sigma I=mr_1^2+mr_2^2+\ldots\ldots\ldots+mr_n^2$$

$$=m\ell^2+m(2\ell)^2+\ldots\ldots+m(n\ell)^2$$

$$\Rightarrow m\ell^2\left[1^2+2^2+\ldots\ldots n^2\right]$$

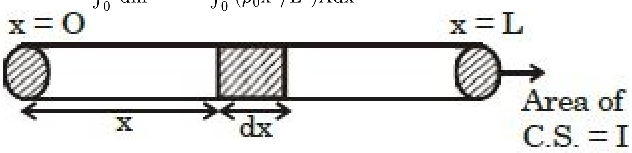
$$=\frac{m\ell^2}{6}n(n+1)(2n+1)$$

20. The variation of density of a cylindrical thick and long rod, is $\rho=\rho_0\frac{x^2}{L^2}$, then position of its centre of mass from $x=0$ end is

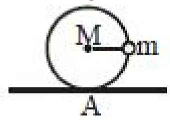
A) $2L/3$ B) $L/2$ C) $L/3$ D) $3L/4$

Solution : (Correct Answer: D)

$$x_{cm}=\frac{\int_0^L xdm}{\int_0^L dm}=\frac{\int_0^L x(\rho_0x^2/L^2)Adx}{\int_0^L(\rho_0x^2/L^2)Adx}=\frac{3L^4}{4L^3}=\frac{3L}{4}$$



21. A uniform body of mass M radius R has a small mass m attached at edge as shown in the figure. The system is placed on a perfectly rough horizontal surface such that mass m is at the same horizontal level as the centre of body. It is assumed that there is no slipping at point A. If I_A is the moment of the inertia of combined system about point of contact A then the normal reaction at point A just after the system is released from rest is .... N. ($M=6\ kg$, $m=2\ kg$, $I_A=4\ kg\ m^2$, $R=1\ m$, $g=10\ m/s^2$)



A) 60 B) 80 C) 75 D) 70

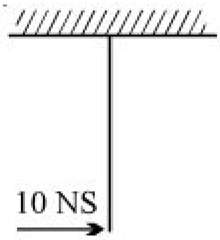
Solution : (Correct Answer: D)

22. If a person sitting on a rotating stool with his hands outstretched, suddenly lowers his hands, then his

A) Kinetic energy will decrease
B) Moment of inertia will decrease
C) Angular momentum will increase
D) Angular velocity will remain constant

Solution : (Correct Answer: B)

23. A thin uniform straight rod of mass $2\ kg$ and length $1\ m$ is free to rotate about its upper end when at rest. It receives an impulsive blow of 10 Ns at its lowest point, normal to its length as shown in figure. The kinetic energy of rod just after impact is .... J



A) 75 B) 100 C) 200 D) none

Solution : (Correct Answer: A)

Angular impulse $=$ change in angular momentum

$$10\times1=I\omega-0$$

$I=$ moment of inertia of rod about it upper end

$$=\frac{1}{12}mL^2+m\left(\frac{L}{2}\right)^2=\frac{1}{3}mL^2$$

$$\frac{1}{3}mL^2\omega=10$$

kinetic energy will be(KE) $=\frac{1}{2}Iw^2$

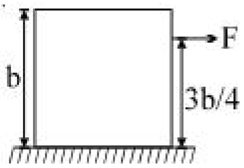
$$KE=\frac{1}{2}\times\left(\frac{1}{3}mL^2\right)\times\left(\frac{30}{mL^2}\right)^2$$

$$KE=\frac{300}{2mL^2}$$

$$KE=\frac{150}{2}$$

$KE = 75J$

24. A uniform cube of side 'b' and mass M rest on a rough horizontal table.A horizontal force F is applied normal to one of the face at a point, at a height $3b/4$ above the base. What should be the coefficient of friction (μ) between cube and table so that is will tip about an edge before it starts slipping?



A) $\mu > \frac{2}{3}$ B) $\mu > \frac{1}{3}$
C) $\mu > \frac{3}{2}$ D) none

Solution : (Correct Answer: A)

$\mathrm{mg}\frac{\mathrm{b}}{2} = \mathrm{F}\frac{3\mathrm{b}}{4}$

$\mathrm{F} = \mathrm{f} < \mu\mathrm{mg} \ldots (2)$

From equation (1) and (2)

$\mathrm{F} < \mu\mathrm{mg}$

$\frac{2mg}{3} < \mu g$

or $\mu > \frac{2}{3}$

25. A solid uniform disk of mass m rolls without slipping down a fixed inclined plane with an acceleration a. The frictional force on the disk due to surface of the plane is :

A) $2\,ma$ B) $3/2\,ma$
C) ma D) $1/2\,ma$

Solution : (Correct Answer: D)

Here as the sphere is rolling down an inclined plane it will produce a torque $\tau = I\alpha$

where $1 =$ moment of inertia and $\alpha =$ angular acceleration

Now the frictional force $f = N\mu_s$

Where $\mathrm{N} =$ normal force $= mg\cos\theta$ and $\mu_s =$ static friction

We also know moment of inertia of sphere $I = \frac{1}{2}mR^2$ 'where Reradius of sphere.

and $\alpha = \frac{\text{acceleration}(a)}{R}$

In the given case torque τ = frictional force $f \times R$

$fR = I\tau$

$fR = \frac{1}{2}mR^2\frac{a}{R}$

$f = \frac{ma}{2}$

26. Two semicircular rings of linear mass densities λ and 3λ and of radius R each are joining to form a complete ring. The distance of the centre of the mass of complete ring from its geometrical centre is

A) $\frac{2R}{3\pi}$ B) $\frac{R}{\pi}$
C) $\frac{2R}{\pi}$ D) $\frac{R}{3\pi}$

Solution : (Correct Answer: B)

Let the mass of ring having density λ be M.

Centre of mass

$\frac{M_1x_1+M_2x_2}{M_1+M_2} = \frac{-M\frac{2R}{\pi}+3M\cdot\frac{2R}{\pi}}{4M} = \frac{R}{\pi}$

27. Two spheres of masses $2M$ and M are initially at rest at a distance R apart. Due to mutual force of attraction they approach each other. When they are at separation $R/2$, the acceleration of the centre of mass of sphere would be .... gm/s^2

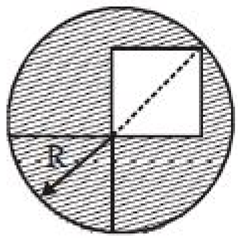
A) 0 B) 1 C) 3 D) 12

Solution : (Correct Answer: A)

The acceleration of center of mass is zero.

As the motion is due to mutual attraction the center of mass will not change.

28. From a circular disc of radius R, a square is cut out with a radius as its diagonal. The center of mass of remainder part is at a distance (from the centre)



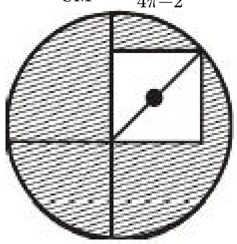
A) $\frac{R}{(4\pi-2)}$ B) $\frac{R}{2\pi}$
C) $\frac{R}{(\pi-2)}$ D) $\frac{R}{(2\pi-2)}$

Solution : (Correct Answer: A)

$\mathrm{X_{CM}} = \frac{\mathrm{M_1x_1-M_2x_2}}{\mathrm{M_1-M_2}}$

$\mathrm{X_{CM}} = \frac{0-1/2\mathrm{r}^2(\mathrm{r}/2)(\sigma)}{\sigma\pi r^2-\mathrm{r}^2/2}$

$\Rightarrow \mathrm{X_{CM}} = \frac{\mathrm{r}}{4\pi-2}$



mass $\propto$ area

29. A particle is moving along a straight line with increasing speed. Its angular momentum about a fixed point on this line

A) Goes on increasing
B) Goes on decreasing
C) May be increasing or decreasing depending on direction of motion
D) Remains zero

Solution : (Correct Answer: D)

$\mathrm{L}=0$ because angle between $\overrightarrow{\mathrm{r}} \& \overrightarrow{\mathrm{v}}$ is always

zero (as point lies on the same line)

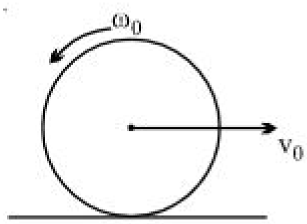
30. Suppose all the people in the world line up at the equator and all start running at speed v_{rel} relative to surface of earth along equatorial circle. Initial angular velocity of earth $=\omega_0$. Moment of inertia of earth I_E, moment of inertia of all people $=I_P$, radius of earth $=R$:

A) There will be no change in angular velocity of rotation of earth.
B) If people run due east change in angular velocity of earth will be $\omega_0-\frac{I_p v_{rel}}{(I_p+I_E)R}$
C) If people run due west change in angular velocity of earth will be $\omega_0-\frac{I_p v_{rel}}{(I_p+I_E)R}$
D) If people run due west change in angular velocity of earth will be $\omega_0-\frac{I_p v_{rel}}{(I_p-I_E)R}$

Solution : (Correct Answer: B)

Apply conservation of angular momentum L_{final} $=L_{initial}$

31. A uniform circular disc placed on a rough horizontal surface has initially a velocity v_0 and an angular velocity ω_0 as shown in the figure. The disc comes to rest after moving some distance in the direction of motion. Then $\frac{v_0}{r\omega_0}$ is



A) $\frac{1}{2}$ B) 1 C) $\frac{3}{2}$ D) 2

Solution : (Correct Answer: A)

Slipping will take place and to oppose the relative motion between the disc and the ground, friction will act in the shown direction.

So we can write $f=ma$ So $a=\frac{f}{m}$

$f\cdot r=I\alpha$

For the disc $I=\frac{1}{2}mr^2$

$\alpha=\frac{2f}{mr}$

If the disc stops moving after travelling for a while, both v_0 and ω_0 become zero at the same time.

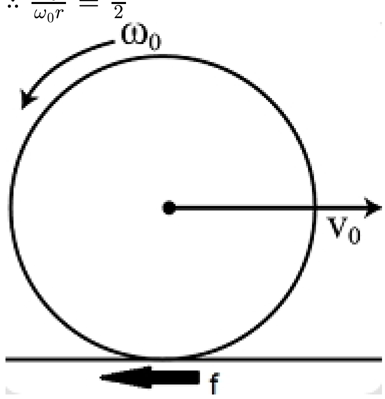
If time t required to stop, we can write

$v_0-at=0$ and $\omega_0-\alpha t=0$

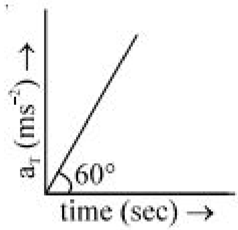
$\therefore \frac{v_0}{a}=\frac{\omega_0}{\alpha}$

$\therefore \frac{v_0 m}{f}=\frac{\omega_0 rm}{2f}$

$\therefore \frac{v_0}{\omega_0 r}=\frac{1}{2}$



32. Tangential acceleration of a particle moving in a circle of radius $1\ m$ varies with time t as (initial velocity of particle is zero). Time after which total acceleration of particle makes and angle of 30^o with radial acceleration is



A) $4\,sec$ B) $4/3\,sec$
C) $2^{2/3}\,sec$ D) $\sqrt{2}$ sec

Solution : (Correct Answer: C)

using equation for a straight line $(y=mx+c)$ where,

$m=\tan 60^{\circ}=\sqrt{3}(\text{ slop })C=0$ from graph

relation b/w a_T and t is obtained as: $-$

$a_T=\tan 60^{\circ} t+0$

this tangential acceleration increases velocity (v) of the particless as.

$a_T=\frac{dv}{dt} \Rightarrow \frac{dv}{dt}=\sqrt{3}t$

$\Rightarrow \int dv=\int \sqrt{3}+dt \Rightarrow v=\frac{\sqrt{3}t^2}{2}$

so, a_T a particular time, centripetal accin (a_c) is given by : $-$

$a_c = \frac{v^2}{r} = \frac{3}{4}t^4(r=1)\vec{a}_T$

let a time $\vec{a}$ masses angle 30° with $\vec{a}_c$. so it makes angles 60° with $\vec{a}_T$ $(\vec{a_c} K \vec{a_T}$ *are perpendicular*$)$

$\Rightarrow \vec{a_T} = \sqrt{3t^2 + \frac{9}{16} + 8\frac{1}{2}}$ that given

$\Rightarrow 3t^2 = \frac{1}{11}\left(3t^2 + \frac{9}{16} + 8\right)$

$\Rightarrow t = 0$ or $t = 2^{2/3}$

33. A rope is wound round a hollow cylinder of mass $5\ kg$ and radius $0.5\ m$. .... rad/s^2 is the angular acceleration of the cylinder if the rope is pulled with a force of $20\,N$?

A) 4 B) 5 C) 6 D) 8

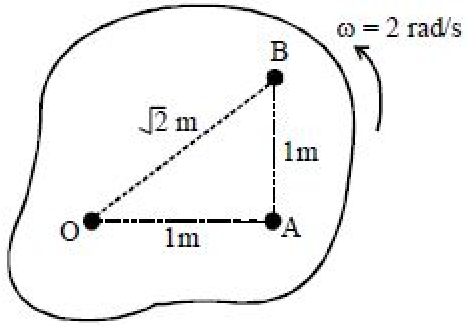
Solution : (Correct Answer: D)

$\tau = \mathrm{I}\alpha = \mathrm{Mr}^2\alpha = \mathrm{Fr}$

$\Rightarrow (5)(0.5)^2\alpha = (20)(0.5)$

$\alpha = 8\mathrm{rad/s}^2$

34. A rigid lamina is rotating about an axis passing perpendicular to its plane through point O as shown in figure. The angular velocity of point B w.r.t A is .... rad/s.



A) 1 B) 2 C) 3 D) 4

Solution : (Correct Answer: B)

In a rigid body, angular velocity of any point on the rigid body w.r.t any other point on the rigid body is constant and equal to angular velocity of rigid body

35. If a force $10\hat{i} + 15\hat{j} + 25\hat{k}$ acts on a system and gives an acceleration $2\hat{i} + 3\hat{j} - 5\hat{k}$ to the centre of mass of the system, the mass of the system is

A) $5\,units$
B) $\sqrt{38}\,units$
C) $5\sqrt{38}\,units$
D) given data is not correct

Solution : (Correct Answer: D)

$\vec{\mathrm{F}}_{\text{ext}} = \mathrm{M}\vec{\mathrm{a}}_{\text{CM}}$

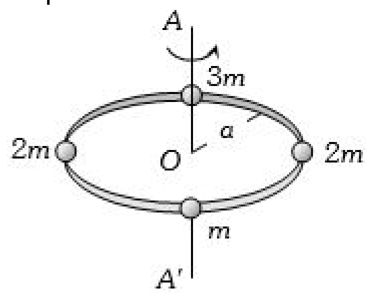
i.e., a_{CM} lies in the direction of $\overrightarrow{\mathrm{F}_{\text{ext}}}$

Here, $\vec{\mathrm{F}}_{\text{ext}} = 5(2\hat{\mathrm{i}} + 3\hat{\mathrm{j}} + 5\hat{\mathrm{k}})$

$\vec{\mathrm{a}}_{\text{CM}} = 2\hat{\mathrm{i}} + 3\hat{\mathrm{j}} - 5\hat{\mathrm{k}}$

since, $\vec{\mathrm{F}}_{\text{ext.}}$ and $\overrightarrow{\mathrm{a}_{cm}}$ are not lying in the same direction, given data is incorrect.

36. Four masses are joined to a light circular frame as shown in the figure. The radius of gyration of this system about an axis passing through the centre of the circular frame and perpendicular to its plane would be



A) $a/\sqrt{2}$ B) $a/2$ C) a D) $2a$

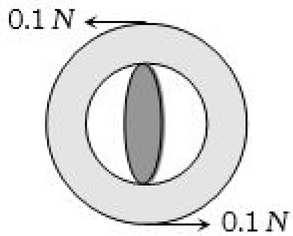
Solution : (Correct Answer: C)

Since the circular frame is massless so we will consider moment of inertia of four masses only.

$I = ma^2 + 2ma^2 + 3ma^2 + 2ma^2 = 8ma^2$ ...(i)

Now from the definition of radius of gyration $I = 8mk^2$ .....(ii) comparing (i) and (ii) radius of gyration $k = a$.

37. Part of the tuning arrangement of a radio consists of a wheel which is acted on by two parallel constant forces as shown in the fig. If the wheel rotates just once, the work done will be about (diameter of the wheel = $0.05\ m$)



A) $0.062\ J$ B) $0.031\ J$
C) $0.015\ J$ D) $0.057\ J$

Solution : (Correct Answer: B)

Rotational kinetic energy $\frac{1}{2}I\omega^2$

$= \frac{1}{2}\left(\frac{1}{2}MR^2\right)\omega^2$

$= \frac{1}{2}\left(\frac{1}{2} \times 10 \times (0.5)^2\right)(20)^2$

$= 250\ J$

38. A body is rolling without slipping on a horizontal plane. If the rotational energy of the body is 40% of the total kinetic energy, then the body might be

A) Cylinder B) Hollow sphere

C) Solid cylinder D) ring

Solution : (Correct Answer: B)

$\frac{\frac{1}{2}I\omega^2}{\frac{1}{2}mv^2} = \frac{40}{60}$

$v = R\omega$

hence $I = \frac{2}{3}mR^2$

$Hollow\ sphere$

39. Two objects of masses $200\ gm$ and $500\ gm$ possess velocities $10\hat{i}m/s$ and $3\hat{i}\ +5\hat{j}m/s$ respectively. The velocity of their centre of mass in m/s is

A) $5\hat{i} - 25\hat{j}$ B) $\frac{5}{7}\hat{i} - 25\hat{j}$
C) $5\hat{i} + \frac{25}{7}\hat{j}$ D) $25\hat{i} - \frac{5}{7}\hat{j}$

Solution : (Correct Answer: C)

Mass of A and B

$M_A = 0.2\text{kg}$ and $M_B = 0.5\text{kg}$

Velocity of A and B

From the conservation of momentum

$M_AV_A + M_BV_B = M_{total}V_{cm}$

$V_{\sigma m} = \frac{M_AV_A+M_BV_B}{M_{total}} = \frac{0.2(10\hat{i})+0.5(3\hat{i}+5\hat{j})}{0.2+0.5}$

$V_{cm} = 5\hat{\imath} + \frac{25}{7}\hat{\jmath}$

Velocity of center of mass $5\hat{\imath} + \frac{25}{7}\hat{j}$

40. A man spinning in free space changes the shape of his body, $eg.$ by spreading his arms or curling up. By doing this, he can change his

A) moment of inertia
B) rotational kinetic energy
C) angular velocity
D) All of the above

Solution : (Correct Answer: D)

Angular momentum remain conserved $\&$ moment of inertia increases, from

$L = I\omega, \omega$ will decrease and

$KE_{\text{Rotational}} = \frac{L^2}{2I}$, so, $KE_{Rotational}$ will increase

41. On a solid sphere lying on a horizontal surface a force F is applied at a height of $R/2$ from the centre of mass. The initial acceleration of a point at the top of the sphere is (there is no slipping at any point)

A) $\frac{15F}{7M}$ B) $\frac{15F}{14M}$
C) $\frac{30F}{7M}$ D) $\frac{F}{M}$

Solution : (Correct Answer: A)

42. The angular velocity of a body is $\vec{\omega} = 2\hat{i} + 3\hat{j} + 4\hat{k}$ and a torque $\vec{\tau}\ = \hat{i}\ + 2\hat{j}\ + 3\hat{k}$ acts on it. The rotational power will be .... W

A) 20 B) 15 C) $\sqrt{17}$ D) $\sqrt{14}$

Solution : (Correct Answer: A)

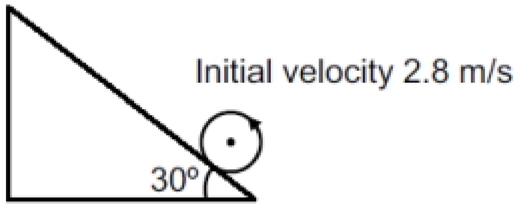
Power $(P) = \vec{\tau} \cdot \vec{\omega}$

$= (i + 2\hat{j} + 3\hat{k}) \cdot (2\hat{i} + 3\hat{j} + 4\hat{k})$

$= 2 + 6 + 12$

$= 20\ W$

43. A sphere pure rolls on a rough inclined plane with initial velocity $2.8\ m/s$. Find the maximum distance on the inclined plane. (in m)



A) 2.74 B) 5.48 C) 1.38 D) 3.2

Solution : (Correct Answer: A)

The acceleration of a sphere down the inclined plane is given

by,

$a = \frac{g\sin\theta}{1+\frac{k^2}{r^2}}$

The moment of inertia of the sphere is given by,

$I = \frac{2}{5}mr^2$

$= mk^2$

Consider above,

$\frac{k^2}{r^2} = \frac{2}{5}$

Use the equation,

$v^2 = 2as \ ...(I)$

Substitute 2.8 for v, $\frac{2}{5}$ for $\frac{k^2}{r^2}$, 30° for θ and $\frac{g\sin\theta}{1+\frac{k^2}{r^2}}$ for a

in

equation (I)

$2.8^2 = 2\left(\frac{g\sin 30^\circ}{1+\frac{2}{5}}\right)s$

$= 2\left(\frac{2(10)(\frac{1}{2})}{\frac{7}{2}}\right)$

$s = 2.8^2\left(\frac{7}{20}\right)$

$= 2.744m$

44. Which of the following is true about the angular momentum of a cylinder down a slope without slipping

A) its magnitude changes but the direction remains same

B) both magnitude and direction change

C) only the direction change

D) neither change

Solution : (Correct Answer: A)

As axis of rotation is along the length of the cylinder are remain same, but speed increases continuously

45. If a solid sphere of mass $1\ kg$ and radius $0.1\ m$ rolls without slipping at a uniform velocity of $1\ m/s$ along a straight line on a horizontal floor, the kinetic energy is

A) $\frac{7}{5}\ J$ B) $\frac{2}{5}\ J$

C) $\frac{7}{10}\ J$ D) $1\ J$

Solution : (Correct Answer: C)

When a body rolls over a smoooth surface, it has linear$K.E.$ and rotational$K.E$

$\therefore E = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$

$where\, \omega = \frac{V}{r}\ and\, I = \frac{2}{5}mr^2\ for\, solid\, sphere.$

$\therefore K.E. = \frac{1}{2}mv^2 + \frac{1}{2}\left(\frac{2}{5}mr^2\right).\frac{V^2}{r^2}$

$= \frac{1}{2}mv^2 + \frac{1}{5}mv^2 = \frac{7}{10}mv^2 = \frac{7}{10} \times 1 \times 1^2$

$= \frac{7}{10}J$