

Physics - Section A (MCQ)

1. Imagine a system of unit in which the unit of mass is 10 kg , length is 1 km and time is 1 min . Then, 1 J in this system is equal to

- 1) 360 2) 3.6
3) 36×10^5 4) 36×10^{-5}

Solution : (Correct Answer: 4)

As joule is the SI unit of energy and dimensional formula for energy is $[ML^2 T^{-2}]$. In this question, we have to convert one unit into another, therefore

$$\begin{aligned} \eta_1 [M_1 L_1^2 T_1^{-2}] &= \eta_2 [M_2 L_2^2 T_2^{-2}] \\ \Rightarrow \eta_2 &= \eta_1 \left[\frac{M_1}{M_2} \right]^1 \left[\frac{L_1}{L_2} \right]^2 \left[\frac{T_1}{T_2} \right]^{-2} \\ &= 1 \times \left[\frac{10 \text{ kg}}{10 \text{ kg}} \right]^1 \left[\frac{1 \text{ km}}{1 \text{ km}} \right]^2 \left[\frac{1 \text{ min}}{1 \text{ min}} \right]^{-2} \\ &= 1 \times \left[\frac{10 \text{ kg}}{10 \text{ kg}} \right]^1 \left[\frac{1 \text{ m}}{10^3 \text{ m}} \right]^2 \left[\frac{1 \text{ s}}{60 \text{ s}} \right]^{-2} \\ &= 1 \times 10^{-1} \times 10^{-6} \times 36 \times 10^2 \\ \eta_2 &= 36 \times 10^{-5} \text{ J} \end{aligned}$$

2. The time dependence of a physical quantity P is given by $P = P_0 e^{\alpha(-\alpha t^2)}$, where α is a constant and t is time. The constant α

- 1) is a dimensionless
2) has dimensions of P
3) has dimensions of T^{-2}
4) has dimensions of T^2

Solution : (Correct Answer: 3)

Given, $P = P_0 \exp(-\alpha t^2)$

As P and P_0 have the same units, therefore αt^2 must be dimensionless for which

$$\alpha = \frac{1}{t^2} = T^{-2}$$

3. The distance travelled by a particle starting from rest and moving with an acceleration $\frac{4}{3} \text{ ms}^{-2}$, in the third second is:

- 1) 6 m 2) 4 m 3) $\frac{10}{3} \text{ m}$ 4) $\frac{19}{3} \text{ m}$

Solution : (Correct Answer: 3)

Distance travelled in the n th second is given by $d_n = u + \frac{a}{2}(2n-1)$

$$\text{put } u = 0, a = \frac{4}{3} \text{ ms}^{-2}, n = 3$$

$$\therefore d = 0 + \frac{4}{3 \times 2}(2 \times 3 - 1) = \frac{4}{6} \times 5 = \frac{10}{3} \text{ m}$$

4. The horizontal range of a projectile is $4\sqrt{3}$ times of its maximum height. The angle of projection will be

- 1) 60° 2) 37°
3) 30° 4) 45°

Solution : (Correct Answer: 3)

$$R = 4\sqrt{3}$$

$$\frac{2u^2 \sin \theta \cos \theta}{g} = 4\sqrt{3} \left(\frac{u^2 \sin^2 \theta}{2g} \right)$$

$$\tan \theta = \frac{1}{\sqrt{3}}$$

$$\therefore \theta = 30^\circ$$

5. A person swims in a river aiming to reach exactly on the opposite point on the bank of a river. His speed of swimming is 0.5 m/s at an angle of 120° with the direction of flow of water. The speed of water is

- 1) 1.0 m/s 2) 0.5 m/s
3) 0.25 m/s 4) 0.43 m/s

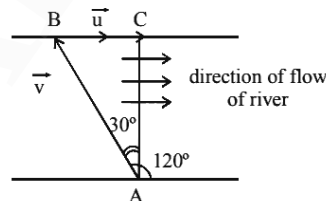
Solution : (Correct Answer: 3)

Let the speed of water = $\vec{u}$

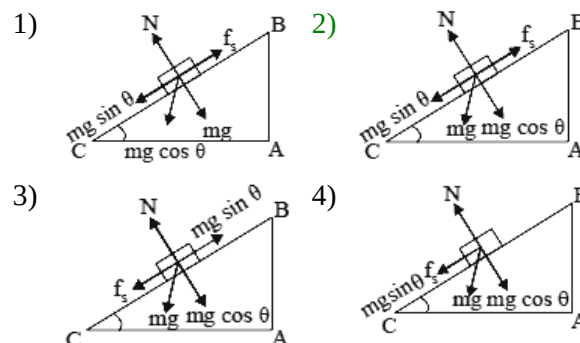
Speed of swimmer = $\vec{v} = 0.5 \text{ m/sec}$

Angle between $\vec{v}$ and $\vec{u}$ is 120° . Then

$$\sin \theta = \frac{\vec{u}}{\vec{v}} \Rightarrow \frac{u}{0.5} = \frac{1}{2} \text{ or } u = 0.25 \text{ ms}^{-1}$$



6. Which figure shows the correct force acting on the body sliding down an inclined plane? ($m \rightarrow$ mass, $f_s \rightarrow$ force of friction)



Solution : (Correct Answer: 2)

If a body slides down, then the force of friction acts upwards along the plane weight(mg) act vertically downwards.

7. Which of the following is a self adjusting force?

- 1) Static friction 2) Limiting friction
3) Dynamic friction 4) Sliding friction

Solution : (Correct Answer: 1)

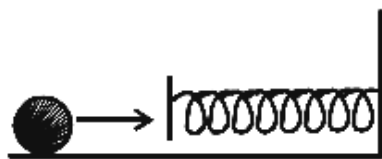
Static friction is a self adjusting force in magnitude and direction.

8. The potential energy of a system increases if work is done
- 1) upon the system by a non conservative force
 - 2) by the system against a conservative force
 - 3) by the system against a non conservative force
 - 4) upon the system by a conservative force

Solution : (Correct Answer: 4)

When work is done upon a system by a conservative force then its potential energy increases.

9. A mass of 0.5 kg moving with a speed of 1.5 m/s on a horizontal smooth surface, collides with a nearly weightless spring of force constant $k = 50 \text{ N/m}$. The maximum compression of the spring would be



- 1) 0.5 m 2) 0.15 m 3) 0.12 m 4) 1.5 m

Solution : (Correct Answer: 2)

Apply law of conservation of energy,

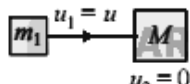
$$\begin{aligned}\frac{1}{2}mv^2 &= \frac{1}{2}kx^2 \\ \Rightarrow mv^2 &= kx^2 \text{ or } 0.5 \times (1.5)^2 = 50 \times x^2 \\ \therefore x &= 0.15 \text{ m}\end{aligned}$$

10. A body of mass 5 kg makes an elastic collision with another body at rest and continues to move in the original direction after collision with a velocity equal to $\frac{1}{10}$ th of its original velocity. Then the mass of the second body is

- 1) 4.09 kg 2) 0.5 kg 3) 5 kg 4) 5.09 kg

Solution : (Correct Answer: 1)

Mass of the first body $m_1 = 5 \text{ kg}$ and for elastic collision coefficient of restitution, $e = 1$.



Let initially body m_1 moves with velocity v after collision velocity becomes $(\frac{u}{10})$.

Let after collision velocity of M block becomes (v_2)

By conservation of momentum

$$m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2 \text{ or } 5u + M \times 0$$

$$= 5 \times \frac{u}{10} + Mv_2$$

$$\text{or } 5u = \frac{u}{2} + Mv_2$$

$$\text{Since, } v_1 - v_2 = -e(u_1 - u_2)$$

$$\text{or } \frac{u}{10} - v_2 = -1(u) \text{ or } \frac{u}{10} + u = v_2$$

$$\text{or } \frac{11u}{10} = v_2$$

Substituting value of v_2 in Eq. (i) from Eq. (ii)

$$5u = \frac{u}{2} + M \left(\frac{11u}{10} \right) = \frac{9 \times 10}{2 \times 11} = \frac{45}{11} \text{ or } 5 - \frac{1}{2}$$

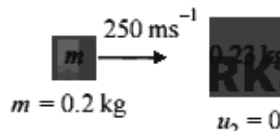
$$= M \left(\frac{11}{10} \right) \Rightarrow M = 4.09 \text{ kg} = 4.09 \text{ kg}$$

11. A bullet of mass 0.02 kg travelling horizontally with velocity 250 ms^{-1} strikes a block of wood of mass 0.23 kg which rests on a rough horizontal surface. After the impact, the block and bullet move together and come to rest after travelling a distance of 40 m. The coefficient of sliding friction of the rough surface is ($g = 9.8 \text{ ms}^{-2}$)

- 1) 0.75 2) 0.61 3) 0.51 4) 0.30

Solution : (Correct Answer: 3)

After collision the bullet and block move together and comes to rest after covering a distance of 40 m.



By conservation of momentum

$$\begin{aligned}m_1u_1 + m_2u_2 &= m_1v_1 + m_2v_2 \\ \Rightarrow 0.02 \times 250 + 0.23 \times 0 &= 0.02v + 0.23v \\ 5 + 0 &= v(0.25) \text{ or } v = 20 \text{ ms}^{-1}\end{aligned}$$

Now, by conservation of energy

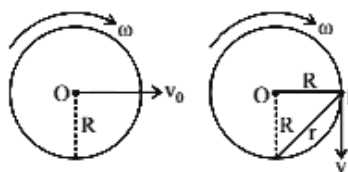
$$\begin{aligned}\text{or } \frac{1}{2}Mv^2 &= \mu R \cdot d \\ \text{or } \frac{1}{2} \times 0.25 \times 400 &= \mu \times 0.25 \times 9.8 \times 40 \\ \Rightarrow \mu &= \frac{200}{9.8 \times 40} = 0.51\end{aligned}$$

12. The centre of a wheel rolling on a plane surface moves with a speed v_0 . A particle on the rim of the wheel at the same level as the centre will be moving at speed

- 1) zero 2) v_0 3) $2v_0$ 4) $\sqrt{2}v_0$

Solution : (Correct Answer: 4)

The situation can be shown as



$$\text{Here } v_0 = R\omega$$

$$\text{At } P, v = r\omega$$

$$= \sqrt{(R^2 + R^2)}\omega = \sqrt{2}R\omega = \sqrt{2}v_0$$

13. The gravitational potential at a place varies inversely with x^2 (i.e., $V = k/x^2$), the gravitational field at that place is

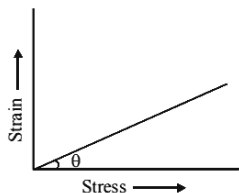
- 1) $2k/x^3$ 2) $-2k/x^3$
3) k/x 4) $-k/x$

Solution : (Correct Answer: 1)

Gravitational intensity,

$$I = -\frac{dv}{dx} = -\frac{d}{dx} \left(\frac{k}{x^2} \right) = \frac{2k}{x^3}$$

14. The value of $\tan(90^\circ - \theta)$ in the graph gives



- 1) Young's modulus of elasticity
- 2) compressibility
- 3) shear strain
- 4) tensile strength

Solution : (Correct Answer: 1)

$$\tan(90^\circ - \theta) = \frac{\text{stress}}{\text{strain}}$$

15. Two liquids of densities d_1 and d_2 are flowing in identical capillary tubes under the same pressure difference. If t_1 and t_2 are time taken for the flow of equal quantities (mass) of liquids, then the ratio of coefficient of viscosity of liquids must be

- 1) $\frac{d_1 t_1}{d_2 t_2}$
- 2) $\frac{t_1}{t_2}$
- 3) $\frac{d_2}{d_1} \frac{t_2}{t_1}$
- 4) $\sqrt{\frac{d_1 t_1}{d_2 t_2}}$

Solution : (Correct Answer: 1)

No Solution Available

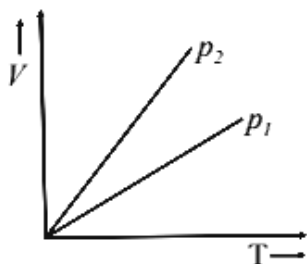
16. A bubble of n mole of helium is submerged at a certain depth in water. The temperature of water increases by $\Delta t^\circ\text{C}$. How much heat is added approximately to helium during expansion?

- 1) $nc_v \Delta t$
- 2) $\frac{nc_p}{\Delta t}$
- 3) $\frac{n^2 c_v}{\Delta t}$
- 4) $nc_p \Delta t$

Solution : (Correct Answer: 2)

$$H = nc_p \Delta t$$

17. In the $V - T$ diagram shown in adjoining figure, what is the relation between p_1 and p_2 ?



- 1) $p_2 = p_1$
- 2) $p_2 < p_1$
- 3) $p_2 > p_1$
- 4) insufficient data

Solution : (Correct Answer: 2)

In an isobaric process, $p = \text{constant}$ Hence, $V \propto T$
i.e., $V = \left(\frac{nR}{p}\right) T$

$\therefore V-T$ graph is a straight line with slope $\propto \frac{1}{p}$

(slope)₂ > (slope)₁ $\therefore p_2 < p_1$

18. If a gas mixture contains 2 moles of O_2 and 4 moles of Ar at temperature T , then what will be the total energy of the system (neglecting all vibrational modes)

- 1) $11RT$
- 2) $15RT$
- 3) $8RT$
- 4) RT

Solution : (Correct Answer: 1)

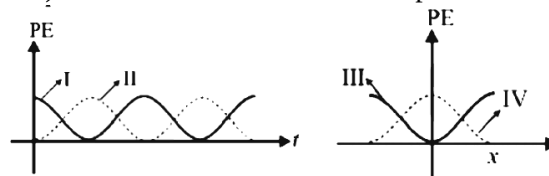
Total energy

$$U = 2\left(\frac{n_1}{2}RT\right) + 4\left(\frac{n_2}{2}RT\right)$$

For O_2 , $n = 5$ and for Ar, $n_2 = 3$

$$\therefore U = 2\left(\frac{5}{2}RT\right) + 4\left(\frac{3}{2}RT\right) = 11RT$$

19. For a particle executing S.H.M. the displacement x is given by $x = A \cos \omega t$. Identify the graph which represents the variation of potential energy (P.E.) as a function of time t and displacement x .



- 1) I, III
- 2) II, IV
- 3) II, III
- 4) I, IV

Solution : (Correct Answer: 1)

In $x = A \cos \omega t$, the particle starts oscillating from extreme position. So at $t = 0$, its potential energy is maximum.

20. Two waves $y_1 = A_1 \sin(\omega t - \beta_1)$ and $y_2 = A_2 \sin(\omega t - \beta_2)$ superimpose to form a resultant wave whose amplitude is

- 1) $A_1 + A_2$
- 2) $|A_1 + A_2|$
- 3) $\sqrt{A_1^2 + A_2^2 - 2A_1 A_2 \sin(\beta_1 - \beta_2)}$
- 4) $\sqrt{A_1^2 + A_2^2 + 2A_1 A_2 \cos(\beta_1 - \beta_2)}$

Solution : (Correct Answer: 4)

Amplitudes A_1 and A_2 are added as vectors. Angle between the two vectors is the phase difference $(\beta_1 - \beta_2)$ between them.

$\therefore$ Resultant wave,

$$R = \sqrt{A_1^2 + A_2^2 + 2A_1 A_2 \cos(\beta_1 - \beta_2)}$$

21. When a metallic plate swings between the poles of a magnet

- 1) no effect on the plate
- 2) eddy currents are set up inside the plate and the direction of the current is along the motion of the plate
- 3) eddy currents are set up inside the plate and the direction of the current oppose the motion of the plate
- 4) eddy currents are set up inside the plate

Solution : (Correct Answer: 3)

When a metallic plate swings between the poles of a magnet, eddy currents are set up inside the plate. These currents set up their own magnetic field which opposes the magnetic field of the poles. Thus, the direction of the current opposes the motion of the plate.

22. The capacitance of a parallel plate capacitor with air as medium is $3\mu\text{F}$. As a dielectric is introduced between the plates, the capacitance becomes $15\mu\text{F}$. The permittivity of the medium in $\text{C}^2 \text{N}^{-1} \text{m}^{-2}$ is

- 1) 8.15×10^{-11} 2) 0.44×10^{-10}
 3) 15.2×10^{12} 4) 1.6×10^{-14}

Solution : (Correct Answer: 2)

Capacitance of air capacitor

$$C_0 = \frac{\epsilon_0 A}{d} = 3\mu\text{F}$$

When a dielectric of permittivity ϵ_r and dielectric constant K is introduced between the plates, then

$$\text{Capacitance, } C = \frac{K\epsilon_0 A}{d} = 15\mu\text{F}$$

Dividing eq. (ii) by (i), we get

$$\frac{C}{C_0} = \frac{d}{\frac{\epsilon_0 A}{d}} = \frac{15}{3}$$

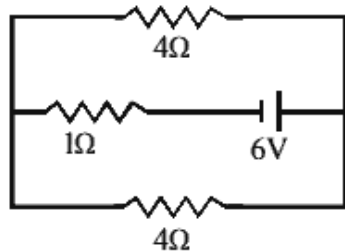
$$\Rightarrow K = 5$$

$\therefore$ permittivity of the medium

$$\epsilon_r = \epsilon_0 K$$

$$= 8.85 \times 10^{-12} \times 5 = 0.44 \times 10^{-10}$$

23. The current in the 1Ω resistor shown in the circuit is



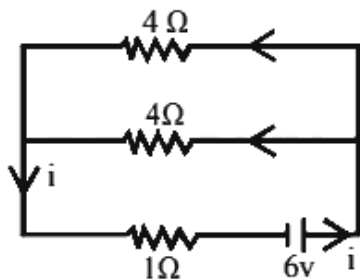
- 1) $\frac{2}{3} \text{ A}$ 2) 3 A 3) 6 A 4) 2 A

Solution : (Correct Answer: 4)

Two 4Ω resistors are in parallel combination. Their equivalent resistance

$$= \frac{4 \times 4}{4 + 4} = \frac{16}{8} = 2\Omega$$

$\therefore$ Total resistance of the



$$\text{network} = 2 + 1 = 3\Omega$$

$\therefore$ Current through 1Ω resistor, $i = \frac{6}{3} = 2 \text{ A}$

24. Radar waves are sent towards a moving airplane and the reflected waves are received. When the airplane is moving towards the radar, the wavelength of the wave

- 1) decrease
 2) increase
 3) remains the same
 4) sometimes increase or decrease

Solution : (Correct Answer: 1)

By Dopplers effect, if source moves towards the observer, frequency received will increase.

$$\text{As } v \propto \frac{1}{\lambda}$$

$\therefore$ wavelength will decrease.

25. The ratio of the amounts of heat developed in the four arms of a balance Wheatstone bridge, when the arms have resistances $P = 100\Omega$, $Q = 10\Omega$, $R = 300\Omega$ and $S = 30\Omega$ respectively is

- 1) $3 : 30 : 1 : 10$ 2) $30 : 3 : 10 : 1$
 3) $30 : 10 : 1 : 3$ 4) $30 : 1 : 3 : 10$

Solution : (Correct Answer: 2)

No Solution

26. The magnetic flux in a closed circuit of resistance 10Ω varies with time as $\phi = (2t - 4t^2 + 1)$. The current in the loop will change its direction after a time of

- 1) 0.25sec 2) 0.5sec 3) 1sec 4) none

Solution : (Correct Answer: 1)

$$e = -\frac{d\phi}{dt} = [-2 + 8t + 0]$$

$$\Rightarrow 8t = 2 \Rightarrow t = \frac{1}{4} \text{ sec} = 0.25 \text{ sec}$$

27. An alternating voltage of 220 V , 50 Hz frequency is applied across a capacitor of capacitance $2\mu\text{F}$. The impedance of the circuit is

- 1) $\frac{\pi}{5000}$ 2) $\frac{1000}{\pi}$
 3) 500π 4) $\frac{5000}{\pi}$

Solution : (Correct Answer: 4)

Impedance of a capacitor is $X_C = 1/\omega C$

$$X_C = \frac{1}{2\pi fC} = \frac{1}{2\pi \times 50 \times 2 \times 10^{-6}} = \frac{5000}{\pi}$$

28. An electromagnetic wave with poynting vector 6Wm^{-2} is absorbed by a surface area 12 m^2 . The force on the surface is

- 1) $24 \times 10^{-8} \text{ N}$ 2) $30 \times 10^{-5} \text{ M}$
 3) $2.4 \times 10^{-12} \text{ N}$ 4) $3.6 \times 10^8 \text{ M}$

Solution : (Correct Answer: 1)

As $F = pA$, but $p = \frac{1}{c}$

$$\text{Force on the surface is given as So, } F = \frac{IA}{C} = \frac{6 \times 12}{3 \times 10^8} = 2.4 \times 10^{-8} \text{ N}$$

29. A prism has a refracting angle of 60° . When placed in the position of minimum deviation, it produces a deviation of 30° . The angle of incidence is

- 1) 30° 2) 45°
 3) 15° 4) 60°

Solution : (Correct Answer: 2)

$$\delta_m = 2i - A$$

$$30^\circ = 2i - 60^\circ$$

$$2i = 30^\circ + 60^\circ = 90^\circ$$

$$i = 45^\circ$$

30. In Young's double slit experiment intensity at a point is $(1/4)$ of the maximum intensity. Angular position of this point is (separation between slits is d)

- 1) $\sin^{-1}(\lambda/d)$
 2) $\sin^{-1}(\lambda/2d)$
 3) $\sin^{-1}(\lambda/3d)$
 4) $\sin^{-1}(\lambda/4d)$

Solution : (Correct Answer: 3)

If a is the amplitude of the wave then

$$\frac{I_{\max}}{4} = a^2 = a^2 + a^2 + 2aa \cos \phi$$

$$\text{or } \cos \phi = -\frac{1}{2}$$

$$\text{or } \phi = \frac{2\pi}{3}$$

Corresponding path difference,

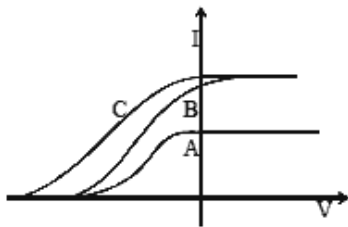
$$\Delta x = \frac{\phi \times \lambda}{2\pi} = \frac{(2\pi/3) \times \lambda}{2\pi} = \frac{\lambda}{3}$$

$$\text{So } d \sin \theta = \frac{\lambda}{3}$$

or

$$\theta = \sin^{-1} \left(\frac{\lambda}{3d} \right)$$

31. In a photoelectric experiment anode potential is plotted against plate current



- 1) A and B will have different intensities while B and C will have different frequencies
 2) B and C will have different intensities while A and C will have different frequencies
 3) A and B will have different intensities while A and C will have equal frequencies
 4) A and B will have equal intensities while B and C will have different frequencies

Solution : (Correct Answer: 1)

From the graph it is clear that A and B have the same stopping potential and therefore the same frequency. Also B and C have the same intensity.

32. An arc of radius r carries charge. The linear density of charge is λ and the arc subtends an angle $\frac{\pi}{3}$ at the centre. What is electric potential at the centre?

- 1) $\frac{\lambda}{4\epsilon_0}$
 2) $\frac{\lambda}{8\epsilon_0}$

- 3) $\frac{\lambda}{12\epsilon_0}$
 4) $\frac{\lambda}{16\epsilon_0}$

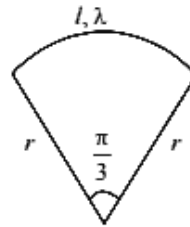
Solution : (Correct Answer: 3)

$$\text{Length of the arc} = r\theta = \frac{r\pi}{3}$$

$$\text{Charge on the arc} = \frac{r\pi}{3} \times \lambda$$

$\therefore$ Potential at centre v

$$= \frac{kq}{r} = \frac{1}{4\pi\epsilon_0} \times \frac{r\pi}{3} \frac{\lambda}{r} = \frac{\lambda}{12\epsilon_0}$$



33. Half-lives of two radioactive substances A and B are respectively 20 minutes and 40 minutes. Initially, the sample of A and B have equal number of nuclei. After 80 minutes, the ratio of remaining number of A and B nuclei is

- 1) 1 : 16 2) 4 : 1 3) 1 : 4 4) 1 : 1

Solution : (Correct Answer: 3)

$$\text{No. of half lives } n_1 = \frac{80}{20} = 4;$$

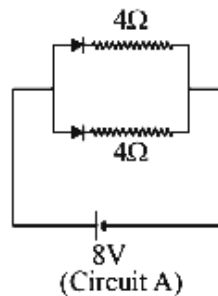
$$n_2 = \frac{80}{40} = 2$$

$$\frac{N_1}{N_{0A}} = \left(\frac{1}{2} \right)^{n_1} = \left(\frac{1}{2} \right)^4 = \frac{1}{16} \Rightarrow \text{for A}$$

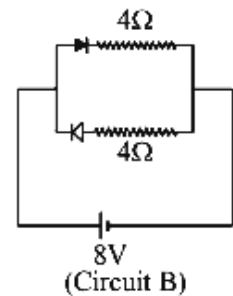
$$\frac{N_2}{N_{0B}} = \left(\frac{1}{2} \right)^{n_2} = \left(\frac{1}{2} \right)^2 = \frac{1}{4} \Rightarrow \text{for B}$$

$$\frac{N_1}{N_2} = \frac{1}{16} \times \frac{4}{1} = \frac{1}{4} \quad (\because N_{0A} = N_{0B})$$

34. Current flowing in each of the following circuit A and B respectively are



(Circuit A)



(Circuit B)

- 1) 1 A, 2 A 2) 2 A, 1 A
 3) 4 A, 2 A 4) 2 A, 4 A

Solution : (Correct Answer: 3)

Here current flows in circuit A as both $(p-n)$ junction diode act as forward biasing.

Total resistance R .

$$\frac{1}{R} = \frac{1}{4} + \frac{1}{4} = \frac{2}{4} \quad \text{or } R = 2\Omega$$

According to Ohm's law

$$V = I_A R$$

$$\text{or } 8 = I_A \times 2 \text{ or } I_A = 4 \text{ A}$$

In circuit B, lower *p* - *n*-junction diode is reverse biased. Hence, no current will flow but upper diode is forward biased so current can flow through it

$$V = I_B R$$

$$\text{or } 8 = I_B \times 4 \text{ or } I_B = 2 \text{ A}$$

35. Beyond which frequency, the ionosphere bands any incident electromagnetic radiation but do not reflect it back towards the earth?

1) 50MHz 2) 40MHz 3) 30MHz 4) 20MHz

Solution : (Correct Answer: 2)

The ionosphere can reflect electromagnetic waves of frequency less than 40MHz but not of frequency more than 40MHz.

Chemistry - Section A (MCQ)

36. Caffeine has a molecular weight of 194u. If it contains 28.9% by mass of nitrogen, number of atom of nitrogen in one molecule of caffeine is

1) 4 2) 6 3) 2 4) 3

Solution : (Correct Answer: 1)

M.wt. of caffeine = 194u % of N present in one molecular of caffeine is 28.9% of

$$194u = \frac{28.9}{100} \times 194 = 56u$$

Mass of one N atom = 14 m

$$= 1 \text{ N atom } (\because 14 \text{ m} = 14u)$$

$$\therefore 56u = \frac{56}{14} \text{ N atom} = 4 \text{ N atom}$$

37. 19.85 mL of 0.1 N NaOH reacts with 20 mL of HCl solution for complete neutralisation. The molarity of HCl solution is

1) 9.9 2) 0.99 3) 0.099 4) 0.0099

Solution : (Correct Answer: 3)

$$\text{Molarity of base} = \frac{\text{normality}}{\text{Acidity}} = \frac{0.1}{1} = 0.1$$

$$M_1 V_1 = M_2 V_2$$

$$0.1 \times 19.85 = M_2 \times 20$$

$$M_2 = 0.09925 \approx 0.099$$

38. Ionisation energy of He^+ is $19.6 \times 10^{-18} \text{ J atom}^{-1}$. The energy of the first stationary state ($n = 1$) of Li^{2+} is

1) $4.41 \times 10^{-16} \text{ J atom}^{-1}$
 2) $-4.41 \times 10^{-17} \text{ J atom}^{-1}$
 3) $-2.2 \times 10^{-15} \text{ J atom}^{-1}$
 4) $8.82 \times 10^{-17} \text{ J atom}^{-1}$

Solution : (Correct Answer: 2)

$$\text{I. } E = \frac{Z^2}{n^2} \times 13.6 \text{ eV} \dots (i)$$

$$\text{or } \frac{I_1}{I_2} = \frac{Z_1^2}{n_1^2} \times \frac{n_2^2}{Z_2^2} \dots (ii)$$

Given $I_1 = -19.6 \times 10^{-18}$, $Z_1 = 2$, $n_1 = 1$, $Z_2 = 3$ and $n_2 = 1$

Substituting these values in equation (ii).

$$-\frac{19.6 \times 10^{-18}}{I_2} = \frac{4}{1} \times \frac{1}{9}$$

$$\text{or } I_2 = -19.6 \times 10^{-18} \times \frac{9}{4} = -4.41 \times 10^{-17} \text{ J/atom}$$

39. The wavelengths of electron waves in two orbits is 3 : 5. The ratio of kinetic energy of electrons will be

1) 25 : 9 2) 5 : 3 3) 9 : 25 4) 3 : 5

Solution : (Correct Answer: 1)

According to de-Broglie's equation.

$$\lambda = \frac{h}{mv} \Rightarrow \lambda^2 = \frac{h^2}{m^2 v^2}$$

$$\text{or } mv^2 = \frac{h^2}{m\lambda^2}$$

$$\therefore \text{KE}(K) = \frac{1}{2} mv^2$$

$$\therefore \text{KE}(K) = \frac{1}{2} \frac{h^2}{m\lambda^2}$$

$$\Rightarrow \frac{K_1}{K_2} = \left(\frac{\lambda_2}{\lambda_1} \right)^2 = \left(\frac{5}{3} \right)^2$$

$$\therefore K_1 : K_2 = 25 : 9$$

40. Which of the following species has highest nucleophilicity?

1) F^- 2) OH^-
 3) CH_3^- 4) NH_2^-

Solution : (Correct Answer: 3)

Lower the electronegativity, higher is the tendency of the atom/group to donate its pair of electrons and hence higher is nucleophilicity. Now among $\text{F}(4.0)$, $\text{O}(3.5)$, $\text{N}(3.0)$ and $\text{C}(2.5)$, C has the lowest electronegativity and hence, CH_3^- has highest nucleophilicity.

41. Which of the following order in respect to ionisation energy is correct?

1) $\text{N} > \text{O} > \text{F}$ 2) $\text{N} < \text{O} < \text{F}$
 3) $\text{N} > \text{O} < \text{F}$ 4) $\text{N} < \text{O} > \text{F}$

Solution : (Correct Answer: 3)

Ionisation energy is the energy required to remove an electron from outer shell of an atom. Since, on moving from left to right in a period, size decreases, thus larger amount of energy is required to remove an electron, i.e. ionisation energy increases.

Although the size of O -atom is smaller as compared to N but the ionisation energy of N is higher.

This is because in case of N, the electron is to be

removed from stable half filled $2p^3$ orbital. Hence, the correct order is $N > O < F$

42. Amongst

NO_3^- , AsO_3^{3-} , CO_3^{2-} , ClO_3^- , SO_3^{2-} and BO_3^{3-} , the non-planar species are

- 1) CO_3^{2-} , SO_3^{2-} and BO_3^{3-}
- 2) AsO_3^{3-} , ClO_3^- and SO_3^{2-}
- 3) NO_3^- , CO_3^{2-} and BO_3^{3-}
- 4) SO_3^{2-} , NO_3^- and BO_3^{3-}

Solution : (Correct Answer: 2)

AsO_3^{3-} , ClO_3^- , and SO_3^{2-} have sp^2 hybridisation and hence are non-planar species, while NO_3^- , CO_3^{2-} and BO_3^{3-} have sp^2 hybridisation and hence are planar species

43. A bubble of air is underwater at temperature 15°C and the pressure 1.5 bar. If the bubble rises to the surface where the temperature is 25°C and the pressure is 1.0 bar, what will happen to the volume of the bubble?

- 1) Volume will become greater by a factor of 1.6
- 2) Volume will become greater by a factor of 1.1
- 3) Volume will become smaller by a factor of 0.70
- 4) Volume will become greater by a factor of 2.9

Solution : (Correct Answer: 1)

(a) $\therefore \frac{p_1 V_1}{T_1} = \frac{p_2 V_2}{T_2}$ (By ideal gas equation) or $\frac{1.5 \times V_1}{288} = \frac{1 \times V_2}{298}$

$\therefore V_2 = 1.55 V_1$
i.e, volume of bubble will be almost 1.6 times to initial volume of bubble.

44. The standard molar heat of formation of ethane, CO_2 and water (1) are respectively -21.1 , -94.1 and -68.3kcal . The standard molar heat of combustion of ethane will be

- 1) -372kcal
- 2) 162kcal
- 3) -240kcal
- 4) 183.5kcal

Solution : (Correct Answer: 1)

Given,

(a) $2\text{C} + 3\text{H}_2 \rightarrow \text{C}_2\text{H}_6$; $\Delta H = -21.1\text{kcal}$

(b) $\text{C} + \text{O}_2 \rightarrow \text{CO}_2$; $\Delta H = -94.1\text{kcal}$

(c) $\text{H}_2 + \frac{1}{2}\text{O}_2 \rightarrow \text{H}_2\text{O}$; $\Delta H = -68.3\text{kcal}$

Now, eqs. $2 \times (b) + 3 \times (c) - (a)$

$\text{C}_2\text{H}_6 + \frac{3}{2}\text{O}_2 \rightarrow 2\text{CO}_2 + 3\text{H}_2\text{O}$

Heat of combustion of Ethane $\Delta x = 2(-94.1)$

$+3(-68.3) - (-21.1)$

$= (-188.2) + (-204.9) - 21.1$

$= -372\text{kcal}$

45. Given that the equilibrium constant for the reaction $2\text{SO}_2(\text{g}) + \text{O}_2(\text{g}) \rightleftharpoons 2\text{SO}_3(\text{g})$ has a value of 278 at a particular temperature. What is the value of the equilibrium constant for the following reaction at the same temperature?

$\text{SO}_3(\text{g}) \rightleftharpoons \text{SO}_2(\text{g}) + \frac{1}{2}\text{O}_2(\text{g})$

- 1) 1.8×10^{-3}
- 2) 3.6×10^{-3}
- 3) 6.0×10^{-2}
- 4) 1.3×10^{-5}

Solution : (Correct Answer: 3)

$2\text{SO}_2 + \text{O}_2 \rightleftharpoons 2\text{SO}_3$ $K = 278$ (given)

$\text{SO}_3 \rightleftharpoons \text{SO}_2 + \frac{1}{2}\text{O}_2$ $K' = \left(\sqrt{\frac{1}{K}}\right)$

$= \sqrt{\frac{1}{278}} = \sqrt{35.97 \times 10^{-4}} = 6 \times 10^{-2}$

46. At 298K , the conductivity of a saturated solution of AgCl in water is $2.6 \times 10^{-6}\text{Scm}^{-1}$. Its solubility product at 298K .

Given : $\lambda^\infty(\text{Ag}^+) = 63.0\text{S cm}^2\text{mol}^{-1}$,

$\lambda^\infty(\text{Cl}^-) = 67.0\text{S cm}^2\text{mol}^{-1}$

- 1) $2.0 \times 10^{-5}\text{M}^2$
- 2) $4.0 \times 10^{-10}\text{M}^2$
- 3) $4.0 \times 10^{-16}\text{M}^2$
- 4) $2 \times 10^{-8}\text{M}^2$

Solution : (Correct Answer: 2)

Solubility $S = \frac{1000k}{\lambda_{\text{AgCl}}^\infty} = \frac{1000 \times 2.6 \times 10^{-6}}{\lambda_{\text{Ag}^+}^\infty + \lambda_{\text{Cl}^-}^\infty}$

$= \frac{2.6 \times 10^{-3}}{63 + 67} = 2 \times 10^{-5}\text{mol L}^{-1}$; $K_{\text{sp}} = S^2$

47. In the reaction, $8\text{Al} + 3\text{Fe}_3\text{O}_4 \rightarrow 4\text{Al}_2\text{O}_3 + 9\text{Fe}$ the number of electrons transferred from the reductant to the oxidant is

- 1) 8
- 2) 4
- 3) 16
- 4) 24

Solution : (Correct Answer: 4)

$8\text{Al} \rightarrow 8\text{Al}^{3+} + 24\text{e}^-$

$9\text{Fe}^{8/3+} + 24\text{e}^- \rightarrow 9\text{Fe}$

Total 24 electrons are transferred.

48. Some statements about heavy water are given below.

- (i) Heavy water is used as moderator in nuclear reactors
- (ii) Heavy water is more associated than ordinary water
- (iii) Heavy water is more effective solvent than ordinary water

Which of the above statements are correct?

- 1) (i) and (ii)
- 2) (i), (ii) and (iii)
- 3) (ii) and (iii)
- 4) (i) and (iii)

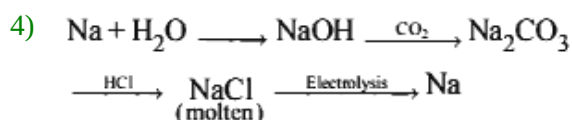
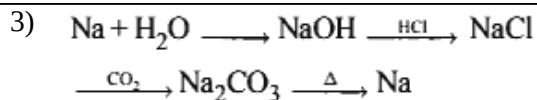
Solution : (Correct Answer: 1)

In nuclear reactors heavy water is used as a moderator. It has higher boiling point as compared to the ordinary water. Thus, it is more associated as compared to ordinary water. The dielectric constant is however higher for H_2O , thus, H_2O is a more effective solvent as compared to heavy water (D_2O).

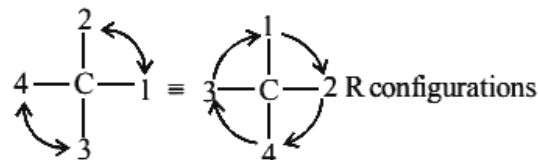
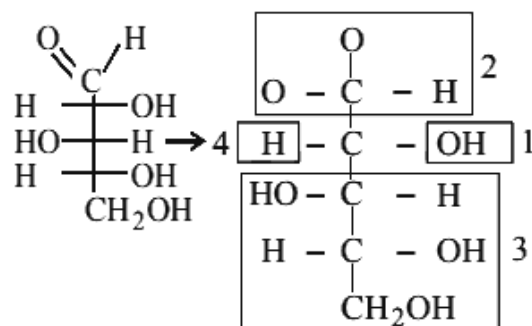
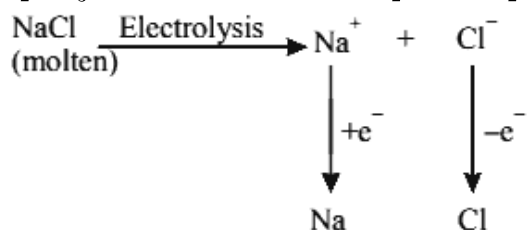
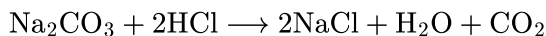
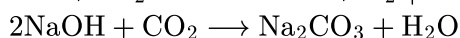
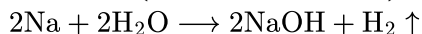
49. Which sequence of reactions shows correct chemical relation between sodium and its compounds?

1) $\text{Na} + \text{O}_2 \rightarrow \text{Na}_2\text{O} \xrightarrow{\text{HCl(aq)}} \text{NaCl} \xrightarrow{\text{CO}_2} \text{Na}_2\text{CO}_3 \xrightarrow{\Delta} \text{Na}$

2) $\text{Na} \xrightarrow{\text{O}_2} \text{Na}_2\text{O} \xrightarrow{\text{H}_2\text{O}} \text{NaOH} \xrightarrow{\text{CO}_2} \text{Na}_2\text{CO}_3 \xrightarrow{\Delta} \text{Na}$



Solution : (Correct Answer: 4)



Arrange the groups in order of priority by following the text.

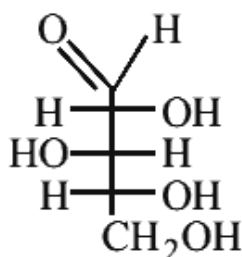
50. Diamond is hard because

- 1) all the four valence electrons are bonded to each carbon atoms by covalent bonds
- 2) it is a giant molecule
- 3) it is made up of carbon atoms
- 4) it cannot be burnt

Solution : (Correct Answer: 1)

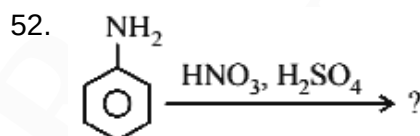
In the structure of diamond each carbon atom in sp^3 hybridised & is covalently bonded with four other carbon atom held at the corners of a regular tetrahedron by covalent bonds. This results in a very big three dimensional polymeric structure in which C - C distance is 154pm and bond angle is 109.5° . Owing to very strong covalent bonds by which atoms are held to gether diamond is the hardest substance known.

51. What is the R and S configuration for each stereogenic centre in this sugar from top to bottom?



- 1) R, R, S
- 2) R, S, S
- 3) R, S, R
- 4) S, S, R

Solution : (Correct Answer: 3)

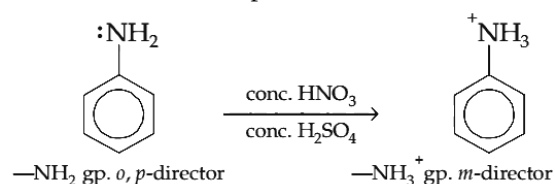


The product P for the above given reaction will be

- 1) *m*-nitroaniline
- 2) *o*-nitroaniline
- 3) *p*-nitroaniline
- 4) both *o* & *p* nitroaniline

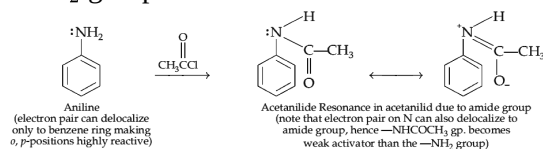
Solution : (Correct Answer: 4)

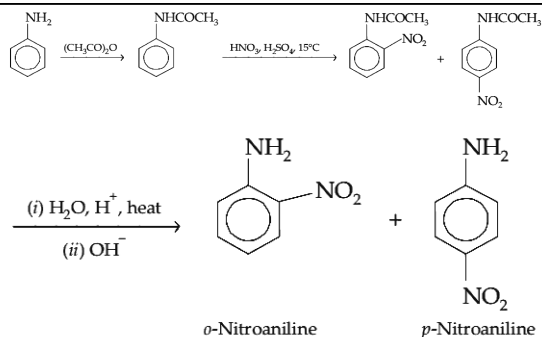
Nitric acid not only nitrates, but also oxidizes the highly reactive ring as well, with loss of much material as dark-coloured tar. Furthermore, in the strongly acidic nitration medium, the amine is converted into anilinium ion ($-\text{NH}_3^+$); substitution is thus controlled not by the $-\text{NH}_2$ group but by the $-\text{NH}_3^+$ group which, because of its positive charge, directs the entering group to the meta- position instead of ortho, and para.



However, all these difficulties are overcome by protecting the amino group by acetylation, with either acetyl chloride or acetic anhydride.

Acetylation ($-\text{NH}_2 \longrightarrow \text{NHCOCH}_3$) converts $-\text{NH}_2$ group to acetamido ($-\text{NHCOCH}_3$) group which is *o*, *p*-directing but lesser activating toward electrophilic aromatic substitution than the parent $-\text{NH}_2$ group.





53. The metal that crystallises in simple cubic system is

- 1) Po 2) Na 3) Cu 4) Ag

Solution : (Correct Answer: 1)

Po-Simple cubic lattice

Na – bcc

Cu – fcc

54. The freezing point of a solution composed of 10.0 g of KCl in 100 g of water is 4.5°C . Calculate the van't Hoff factor, i for this solution.

- 1) 2.50 2) 1.8 3) 1.2 4) 1.3

Solution : (Correct Answer: 2)

$$\Delta T_f = i K_f m$$

$$m = \frac{10 \times 1000}{74.55 \times 100} = 1.34 \text{ m}$$

[Molecular Mass of KCl = 74.55]

$$i = \frac{\Delta T_f}{K_f m} = \frac{4.5^\circ\text{C}}{(1.86^\circ\text{C/m})(1.34 \text{ m})} = 1.8$$

55. The standard Gibbs free energy change, ΔG° is related to equilibrium constant, K_p as

- 1) $K_p = -RT \ln \Delta G^\circ$
 2) $K_p = \left[\frac{e}{RT} \right]^{\Delta G^\circ}$
 3) $K_p = -\frac{\Delta G^\circ}{RT}$
 4) $K_p = e^{-\Delta G^\circ/RT}$

Solution : (Correct Answer: 4)

ΔG° and K_p are related as

$$\Delta G^\circ = -RT \ln K_p$$

$$\Rightarrow \ln K_p = \frac{\Delta G^\circ}{-RT} \Rightarrow K_p = e^{-\Delta G^\circ/RT}$$

56. The rate constant of a reaction at temperature 200 K is 10 times less than the rate constant at 400 K. What is the activation energy of the reaction?

- 1) 1842.4R 2) 460.6R
 3) 230.3R, 4) 921.2R

Solution : (Correct Answer: 3)

At $T_1 = 200 \text{ K}$, if $k_1 = k$

then at $T_2 = 400 \text{ K}$, $k_2 = 10k$

$$\log \frac{10k}{k} = \frac{E_a}{2.303R} \left(\frac{400-200}{400 \times 200} \right)$$

$$E_a = 921.2R$$

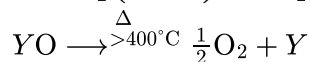
57. Which one of the following is the most effective in causing the coagulation of an As_2S_3 sol?

- 1) KCl 2) AlCl_3
 3) MgSO_4 4) $\text{K}_3\text{Fe}(\text{CN})_6$

Solution : (Correct Answer: 1)

According to Hardy Schulze rule, greater the valency of the coagulating ion, greater is its coagulating power. Thus, out of the given, AlCl_3 (Al^{3+}) is most effective for causing coagulation of As_2S_3 sol.

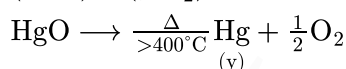
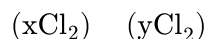
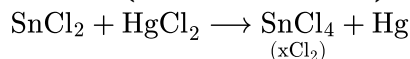
58. XCl_2 (excess) + $\text{YCl}_2 \rightarrow \text{XCl}_4 + \text{Y} \downarrow$



Ore of Y would be,

- 1) siderite 2) malachite
 3) hornsilver 4) cinnabar

Solution : (Correct Answer: 4)

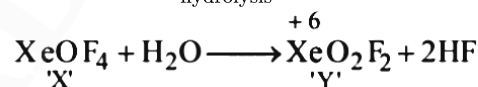
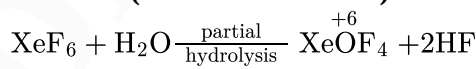


So ore of Y is HgS i.e., Cinnabar.

59. Xenon hexafluoride on partial hydrolysis produces compounds 'X' and 'Y'. Compounds 'X', 'Y' and the oxidation state of Xe are respectively:

- 1) XeOF_4 (+6) and XeO_3 (+6)
 2) XeO_2 (+4) and XeO_3 (+6)
 3) XeOF_4 (+6) and $\text{XeO}_2 \cdot \text{F}_2$ (+6)
 4) $\text{XeO}_2 \cdot \text{F}_2$ (+6) and XeO_2 (+4)

Solution : (Correct Answer: 3)



60. Which one of the following has maximum number of unpaired electrons?

- 1) Mg^{2+} 2) Ti^{3+}
 3) V^{3+} 4) Fe^{2+}

Solution : (Correct Answer: 4)

$\text{Mg}^{2+} = 1 \text{ s}^2, 2 \text{ s}^2, 2 \text{ p}^6$ (No unpaired electrons)

$\text{Ti}^{3+} = 1 \text{ s}^2, 2 \text{ s}^2, 2 \text{ p}^6, 3 \text{ s}^2, 3 \text{ p}^6, 4 \text{ s}^0, 3 \text{ d}^1$ (One unpaired electrons)

$\text{V}^{3+} = 1 \text{ s}^2, 2 \text{ s}^2, 2 \text{ p}^6, 3 \text{ s}^2, 3 \text{ p}^6, 4 \text{ s}^0, 3 \text{ d}^2$ (Two unpaired electrons)

$\text{Fe}^{2+} = 1 \text{ s}^2, 2 \text{ s}^2, 2 \text{ p}^6, 3 \text{ s}^2, 3 \text{ p}^6, 4 \text{ s}^0, 3 \text{ d}^6$ (Four unpaired electrons)

$\therefore \text{Fe}^{2+}$ has highest number of unpaired electrons.

61. AgCl dissolves in a solution of NH_3 but not in water because

- 1) NH_3 is a better solvent than H_2O
 2) Ag^+ forms a complex ion with NH_3
 3) NH_3 is a stronger base than H_2O
 4) the dipole moment of water is higher than NH_3

Solution : (Correct Answer: 2)

AgCl dissolves in a solution of NH_3 but not in water because Ag^+ forms soluble complex ion with NH_3 .

62. $\text{CH}_3\text{CH}_2\text{Br}$ undergoes Wurtz reaction. We may expect some of the following product

A : $\text{CH}_3\text{CH}_2\text{CH}_2\text{CH}_3$

B : $\text{CH}_2 = \text{CH}_2$

C : $\text{CH}_3 - \text{CH}_3$

Select correct product.

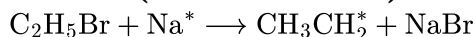
1) Only A

2) A and B

3) A, B and C

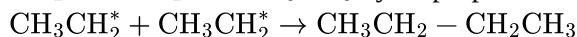
4) A and C

Solution : (Correct Answer: 3)

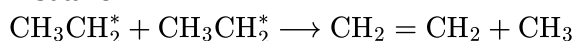


Intermediate free radical CH_3CH_2^* combines to form $\text{CH}_3\text{CH}_2 - \text{CH}_2\text{CH}_3$ (as a main product) and also

$\text{CH}_2 = \text{CH}_2$ and CH_3CH_3 by disproportionation



n-butane



– CH_3

(disproportion reaction)

63. Ethanol and dimethyl ether form a pair of functional isomers. The boiling point of ethanol is higher than that of dimethyl ether, due to the presence of

1) H-bonding in ethanol

2) H-bonding in dimethyl ether

3) CH_3 group in ethanol

4) CH_3 group in dimethyl ether

Solution : (Correct Answer: 1)

Due to H-bonding, the boiling point of ethanol is much higher than that of the isomeric diethyl ether.

64. Match the compounds given in List-I with List-II and select the suitable option using the codes given below

List-I	List-II
A. Benzaldehyde	1. Phenolphthalein
B. Phthalic anhydride	2. Benzoin condensation
C. Phenyl benzoate	3. Oil of wintergreen
D. Methyl salicylate	4. Fries rearrangement

Codes:

A B C D

1) 4 1 3 2

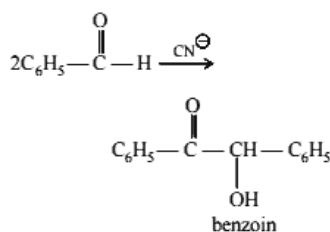
2) 4 2 3 1

3) 2 3 4 1

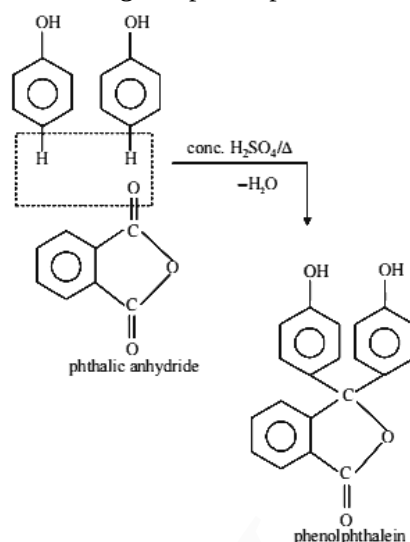
4) 2 1 4 3

Solution : (Correct Answer: 4)

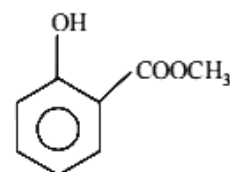
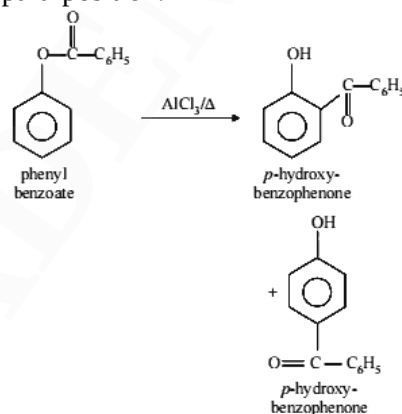
(a) Benzoin condensation : Heating ethanolic solution with strong alkali like KCN or NaCN, benzoin is obtained.



(b) Formation of phenolphthalein phenol is treated with phthalic anhydride in the presence of conc. H_2SO_4 , it gives phenolphthalein, an indicator.

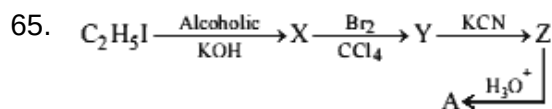


(c) Fries rearrangement Phenyl benzoate heated with anhydrous AlCl_3 in the presence of inert solvent gives ortho-and Para-hydroxybenzophenone. In this rearrangement, there is only a benzoyl group migration from the phenolic oxygen to an ortho-and para-position.



(d) Methylsalicylate

(A chief constituent of oil of wintergreen)



The product 'A' is

1) succinic acid

2) melonic acid

3) oxalic acid

4) maleic acid

Solution : (Correct Answer: 1)



$$2) \frac{x}{1+x} < x < \log(1+x)$$

$$3) x < \log(1+x) < \frac{x}{1+x}$$

$$4) \frac{x}{1+x} < \log(1+x) < x$$

Solution : (Correct Answer: 4)

$$\text{Let } f(x) = \log(1+x) - \frac{x}{1+x}$$

$$\begin{aligned} \therefore f'(x) &= \frac{1}{1+x} - \frac{(1+x) \cdot 1 - x \cdot 1}{(1+x)^2} \\ &= \frac{1}{1+x} - \frac{1}{(1+x)^2} = \frac{x}{(1+x)^2} \end{aligned}$$

which is positive. $[\because x > 0]$

$\therefore f(x)$ is monotonic increasing, when $x > 0$.

$$\Rightarrow f(x) > f(0)$$

$$\text{Now, } f(0) = \log 1 - 0 = 0$$

$$\therefore f(x) > 0$$

$$\Rightarrow \log(1+x) - \frac{x}{1+x} > 0$$

$$\Rightarrow \frac{x}{1+x} < \log(1+x)$$

Also, for $x > 0$,

Also, for $x > 0$,

$$\Rightarrow x^2 > 0 \Rightarrow x^2 + x > x$$

$$\Rightarrow x(x+1) > x$$

$$x > \frac{x}{x+1}$$

From eqs. (i) and (ii), we get

$$\frac{x}{x+1} < \log(1+x) < x$$

$[\because \log(1+x) < x \text{ for } x > 0]$

73. Let $f(x) = x^2 + ax + b$, where $a, b \in \mathbb{R}$. If $f(x) = 0$ has all its roots imaginary, then the roots of $f(x) + f'(x) + f''(x) = 0$ are

- 1) real and distinct 2) imaginary
3) equal 4) rational and equal

Solution : (Correct Answer: 2)

$f(x) = x^2 + ax + b$ has imaginary roots.

$$\therefore \text{Discriminant, } D < 0 \Rightarrow a^2 - 4b < 0$$

$$f'(x) = 2x + a$$

$$\Rightarrow f''(x) = 2$$

$$\text{Also, } f(x) + f'(x) + f''(x) = 0 \dots (i)$$

$$\Rightarrow x^2 + ax + b + 2x + a + 2 = 0$$

$$\Rightarrow x^2 + (a+2)x + b + a + 2 = 0$$

$$\begin{aligned} \therefore x &= \frac{-(a+2) \pm \sqrt{(a+2)^2 - 4(a+b+2)}}{2} \\ &= \frac{-(a+2) \pm \sqrt{a^2 - 4b - 4}}{2} \end{aligned}$$

Since, $a^2 - 4b < 0 \therefore a^2 - 4b - 4 < 0$ Hence, Eq. (i) has imaginary roots.

74. If $\sin \theta, \cos \theta, \tan \theta$ are in G.P. then $\cos^9 \theta + \cos^6 \theta + 3 \cos^5 \theta - 1$ is equal to

- 1) -1 2) 0 3) 1 4) 2

Solution : (Correct Answer: 2)

Given : $\sin \theta, \cos \theta, \tan \theta$ are in G.P.

$$\Rightarrow \cos^2 \theta = \sin \theta \tan \theta \Rightarrow \cos^3 \theta = \sin^2 \theta$$

$$\Rightarrow \cos^3 \theta = 1 - \cos^2 \theta$$

$$\Rightarrow (\cos^3 \theta + \cos^2 \theta) = 1 \dots (1)$$

Cubic both sides, we have

$$\cos^9 \theta + \cos^6 \theta + 3 \cos^5 \theta \cdot (\cos^3 \theta + \cos^2 \theta) = 1$$

$$\Rightarrow \cos^9 \theta + \cos^6 \theta + 3 \cos^5 \theta = 1$$

[Using equation (1)]

$$\Rightarrow \cos^9 \theta + \cos^6 \theta + 3 \cos^5 \theta - 1 = 0$$

75. A complex number z is such that $\arg\left(\frac{z-2}{z+2}\right) = \frac{\pi}{3}$. The points representing this complex number will lie on

- 1) an ellipse 2) a parabola
3) a circle 4) a straight line

Solution : (Correct Answer: 3)

$$\arg\left(\frac{z-2}{z+2}\right) = \frac{\pi}{3}$$

$$\Rightarrow \arg\left(\frac{x-2+iy}{x+2+iy}\right) = \frac{\pi}{3}$$

$$\Rightarrow \arg(x-2+iy) - \arg(x+2+iy) = \frac{\pi}{3}$$

$$\Rightarrow \tan^{-1}\left(\frac{y}{x-2}\right) - \tan^{-1}\left(\frac{y}{x+2}\right) = \frac{\pi}{3}$$

$$\Rightarrow \frac{4y}{x^2+y^2-4} = \sqrt{3}$$

$$\Rightarrow \sqrt{3}(x^2+y^2) - 4y - 4\sqrt{3} = 0$$

which is an equation of a circle.

76. The solution of $(x \operatorname{cosec}\left(\frac{y}{x}\right) - y) dx + x dy = 0$ is

$$1) \log|x| - \cos\left(\frac{x}{y}\right) = c$$

$$2) \log|x| - \cos\left(\frac{y}{x}\right) = c$$

$$3) \log|x| - \sin\left(\frac{x}{y}\right) = c$$

$$4) \log|x| - \sin\left(\frac{y}{x}\right) = c$$

Solution : (Correct Answer: 2)

$$\text{Given : } (x \operatorname{cosec}\left(\frac{y}{x}\right) - y) dx + x dy = 0$$

$$\Rightarrow \left(\frac{x}{\sin\left(\frac{y}{x}\right)} - y\right) dx + x dy = 0$$

$$\Rightarrow [x - y \cdot \sin\left(\frac{y}{x}\right)] dx + x \sin\left(\frac{y}{x}\right) dy = 0$$

$$\Rightarrow \frac{dy}{dx} = \frac{y \sin \frac{y}{x} - x}{x \sin\left(\frac{y}{x}\right)} \dots (1)$$

$$\text{Put } \frac{y}{x} = z \Rightarrow y = zx$$

$$\therefore \frac{dy}{dx} = z \cdot 1 + x \cdot \frac{dz}{dx} = z + x \frac{dz}{dx}$$

$\therefore$ Equation (1) becomes

$$z + x \frac{dz}{dx} = \frac{zx \cdot \sin z - x}{x \sin z} = z - \operatorname{cosec} z$$

$$\Rightarrow x \frac{dz}{dx} = -\operatorname{cosec} z \Rightarrow -\sin z dz = \frac{dx}{x}$$

$$\Rightarrow \log|x| = \cos z + c$$

$$\Rightarrow \log|x| - \cos\left(\frac{y}{x}\right) = c$$

77. A binary sequence is an array of 0's and 1's. The number of n -digit binary sequences which contain even number of 0's is

$$1) 2^{n-1} \quad 2) 2^n - 1$$

$$3) 2^{n-1} - 1 \quad 4) 2^n$$

Solution : (Correct Answer: 1)

The required number of ways = The even number of 0's i.e., $\{0, 2, 4, 6, \dots\}$

$$= \frac{n!}{n!} + \frac{n!}{2!(n-2)!} + \frac{n!}{4!(n-4)!} + \dots$$

$$= {}^nC_0 + {}^nC_2 + {}^nC_4 + \dots = 2^{n-1}$$

78. Five balls of different colours are to be placed in three boxes of different sizes. Each box can hold all five balls. In how many ways can we place the balls so that no box remains empty?

1) 50 2) 100 3) 150 4) 200

Solution : (Correct Answer: 3)

Let the boxes be marked as A, B and C. We have to ensure that no box remains empty and all five balls have to put in. There will be two possibilities:

(i) Any two box containing one ball each and 3rd box containing 3 balls. Number of ways

$$= A(1)B(1)C(3)$$

$$= {}^5C_1 \cdot {}^4C_1 \cdot {}^3C_3 = 5 \cdot 4 \cdot 1 = 20$$

(ii) Any two box containing 2 balls each and third containing 1 ball, the number of ways

$$= A(2)B(2)C(1) = {}^5C_2 \cdot {}^3C_2 \cdot {}^1C_1$$

$$= 10 \times 3 \times 1 = 30$$

Since, the box containing 1 ball could be any of the three boxes A, B, C. Hence, the required number of ways = $30 \times 3 = 90$.

Hence, total number of ways = $20 + 90 = 110$.

79. Sum of first 'n' terms of a series $a_1 + a_2 + \dots + a_n$ is given by $S_n = \frac{n(n^2-1)(n+2)}{4}$, then the value of $\lim_{n \rightarrow \infty} \sum_{r=2}^n \frac{1}{a_r}$ is

1) 4 2) 2 3) $\frac{1}{4}$ 4) $\frac{1}{2}$

Solution : (Correct Answer: 3)

$$\begin{aligned} S_n &= \frac{n(n^2-1)(n+2)}{4} \\ &= \frac{n(n+1)(n-1)(n+2)}{4} \\ &= \frac{n(n+1)}{4} (n^2+n-2) \\ \Rightarrow S_n &= \frac{n(n+1)}{4} (n^2+n) - \frac{n(n+1)}{4} (2) \\ &= \frac{n^2(n+1)^2}{4} - \frac{n(n+1)}{2} \end{aligned}$$

$$\therefore \frac{n^2(n+1)^2}{4} = \Sigma n^3 \text{ and } \frac{n(n+1)}{4} = \Sigma n$$

$$\Rightarrow S_n = \Sigma (n^3 - n)$$

Therefore, $a_n = n^3 - n$

$$\Rightarrow a_r = r^3 - r = r(r^2 - 1) = r(r-1)(r+1)$$

$$\text{and } \sum_{r=2}^n \frac{1}{a_r} = \sum_{r=2}^n \frac{1}{r(r-1)(r+1)}$$

$$= \frac{1}{2} \left(\frac{1}{2} - \frac{1}{n(n+1)} \right) \text{ and } \lim_{n \rightarrow \infty} \sum_{r=2}^n \frac{1}{a_r} = \frac{1}{2} \left(\frac{1}{2} - 0 \right) = \frac{1}{4}$$

80. Let S be a finite set containing n elements. Then the total number of commutative binary operation on S is

1) $n^{\lceil \frac{n(n+1)}{2} \rceil}$ 2) $n^{\lceil \frac{n(n-1)}{2} \rceil}$
3) $n^{(n^2)}$ 4) $2^{(n^2)}$

Solution : (Correct Answer: 1)

Let $S = \{a_i\}$ where $i = 1, 2, \dots, n$. From commutative operations,

$a_i * a_j = a_j * a_i \dots$ (i) $\forall i, j = 1, 2, 3 \dots n$ where * represents a binary operation

$\therefore$ Number of distinct elements in $S \times S$

i.e., $\{a_i\} \times \{a_j\}$ subject to the condition (i)
 $i=1, 2, \dots, n \quad j=1, 2, \dots, n$

$$= n \{(a_1, a_1), (a_1, a_2), \dots, (a_1, a_n)$$

$$(a_2, a_2), (a_2, a_3), \dots, (a_2, a_n)$$

$$\dots (a_{n-1}, a_{n-1}), (a_{n-1}, a_n), (a_n, a_n)$$

$$= n + (n-1) + (n-2) + \dots + 2 + 1 = \frac{n(n+1)}{2}$$

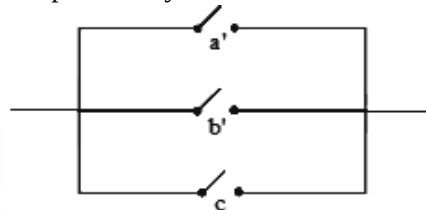
$\therefore$ No. of commutative binary operations

= No. of functions $f: S \times S \rightarrow S$ subject to (i)

$$= n \cdot n \cdot n \dots \frac{n(n+1)}{2} \text{ times} = n^{\frac{n(n+1)}{2}}$$

81. Consider the switching circuit

Logical expression corresponding to the complimentary to the above circuit is



1) $a.b.c'$ 2) $a + b + c'$
3) $a' \cdot b' \cdot c$ 4) none

Solution : (Correct Answer: 1)

Logical expression corresponding to above circuit is

$$= a' + b' + c$$

Logical expression corresponding to the complimentary to the above circuit is

$$(a' + b' + c)' = (a' + b')' \cdot c'$$

$$= (a')' \cdot (b')' \cdot c'$$

$$= a \cdot b \cdot c' \quad [(a')' = a; (b')' = b]$$

82. Find the value of $\sin 12^\circ \sin 48^\circ \sin 54^\circ$.

1) $\frac{1}{2}$ 2) $\frac{1}{4}$ 3) $\frac{1}{6}$ 4) $\frac{1}{8}$

Solution : (Correct Answer: 4)

$$y^2 = 2a(x - a)$$

$$\sin 12^\circ \sin 48^\circ \sin 54^\circ$$

$$= \sin 12^\circ \sin (60^\circ - 12^\circ) \sin (90^\circ - 36^\circ)$$

$$= \frac{\sin 12^\circ \sin (60^\circ - 12^\circ) \sin 72^\circ \cos 36^\circ}{\sin 72^\circ}$$

$$= \frac{[\sin 12^\circ \sin (60^\circ - 12^\circ) \sin (60^\circ + 12^\circ) \cos 36^\circ]}{\sin 72^\circ}$$

$$= \frac{\sin 12^\circ (\sin^2 60^\circ - \sin^2 12^\circ) \cos 36^\circ}{\sin 72^\circ}$$

$$= \frac{\sin 12^\circ \left(\frac{3}{4} - \sin^2 12^\circ \right) \cos 36^\circ}{\sin 72^\circ}$$

$$= \frac{3 \sin 12^\circ - 4 \sin^3 12^\circ}{4} \cdot \frac{\cos 36^\circ}{\sin 72^\circ}$$

$$= \frac{\sin 72^\circ}{24 \cos 36^\circ} = \frac{1}{4} \cdot \frac{1}{8}$$

$$= \frac{\sin 72^\circ}{8}$$

83. $\frac{\cos \theta}{1 - \tan \theta} + \frac{\sin \theta}{1 - \cot \theta}$ is equal to

1) $\sin \theta - \cos \theta$ 2) $\sin \theta + \cos \theta$
3) $\tan \theta + \cot \theta$ 4) $\tan \theta - \cot \theta$

Solution : (Correct Answer: 2)

$$\begin{aligned} & \frac{\cos \theta}{1 - \tan \theta} + \frac{\sin \theta}{1 - \cot \theta} \\ &= \frac{\cos \theta}{1 - \frac{\sin \theta}{\cos \theta}} + \frac{\sin \theta}{1 - \frac{\cos \theta}{\sin \theta}} \\ &= \frac{\cos^2 \theta}{\cos \theta - \sin \theta} - \frac{\sin^2 \theta}{\cos \theta - \sin \theta} \\ &= \frac{\cos^2 \theta - \sin^2 \theta}{\cos \theta - \sin \theta} = \cos \theta + \sin \theta \end{aligned}$$

84. A straight line through the point A(3, 4) is such that its intercept between the axes is bisected at A, its equation is

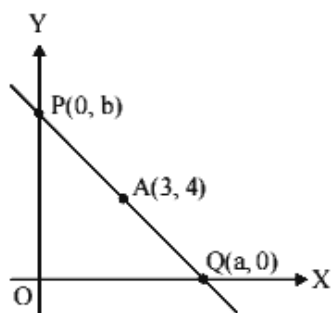
- 1) $3x - 4y + 7 = 0$ 2) $4x + 3y = 24$
3) $3x + 4y = 25$ 4) $x + y = 7$

Solution : (Correct Answer: 2)

A is mid point of line PQ.

$$\therefore 3 = \frac{a+0}{2} \Rightarrow a = 6$$

$$\text{and } 4 = \frac{0+b}{2} \Rightarrow b = 8$$



Thus, equation of line is

$$\begin{aligned} \frac{x}{6} + \frac{y}{8} &= 1 \\ \Rightarrow 4x + 3y &= 24 \end{aligned}$$

85. If $\left| \frac{z-25}{z-1} \right| = 5$, the value of $|z|$
1) 3 2) 4 3) 5 4) 6

Solution : (Correct Answer: 3)

Given that

$$\left| \frac{z-25}{z-1} \right| = 5 \Rightarrow |Z - 25| = 5|Z - 1|$$

Let $Z = x + iy$, then

$$\begin{aligned} |x + iy - 25| &= 5|x + iy - 1| \\ \Rightarrow |(x - 25) + iy| &= 5|(x - 1) + iy| \end{aligned}$$

Squaring both sides, we get

$$\begin{aligned} (x - 25)^2 + y^2 &= 25 \{(x - 1)^2 + y^2\} \\ \Rightarrow x^2 - 50x + 625 + y^2 &= 25x^2 - 50x + 25 + 25y^2 \\ \Rightarrow 24x^2 + 24y^2 - 600 &= 0 \\ \Rightarrow x^2 + y^2 - 25 &= 0 \\ \Rightarrow |x + iy|^2 = 25 &\Rightarrow |Z|^2 = 5^2 \\ \Rightarrow |Z| &= 5 \end{aligned}$$

86. The value of $\int_1^4 |x - 3| dx$ is equal to
1) 2 2) $\frac{5}{2}$ 3) $\frac{1}{2}$ 4) $\frac{3}{2}$

Solution : (Correct Answer: 2)

$$\begin{aligned} \int_1^4 |x - 3| dx &= \int_1^3 -(x - 3) dx + \int_3^4 (x - 3) dx \\ &= -\left[\frac{x^2}{2} - 3x \right]_1^3 + \left[\frac{x^2}{2} - 3x \right]_3^4 \\ &= -\frac{1}{2} [3^2 - 1^2] + 3[3 - 1] + \frac{1}{2} [4^2 - 3^2] - 3[4 - 3] \\ &= -\frac{1}{2} (8) + 3(2) + \frac{1}{2} (7) - 3(1) \\ &= -4 + 6 + \frac{7}{2} - 3 = \frac{5}{2} \end{aligned}$$

87. The normal at the point $(at_1^2, 2at_1)$ on the parabola meets the parabola again in the point $(at_2^2, 2at_2)$, then

- 1) $t_2 = -t_1 + \frac{2}{t_1}$ 2) $t_2 = -t_1 - \frac{2}{t_1}$
3) $t_2 = t_1 - \frac{2}{t_1}$ 4) $t_2 = t_1 + \frac{2}{t_1}$

Solution : (Correct Answer: 2)

Equation of the normal at point $(at_1^2, 2at_1)$ on parabola is

$$y = -t_1 x + 2at_1 + at_1^3$$

It also passes through $(at_2^2, 2at_2)$

$$\begin{aligned} \text{So, } 2at_2 &= -t_1(at_2^2) + 2at_1 + at_1^3 \\ \Rightarrow 2t_2 - 2t_1 &= -t_1(t_2^2 - t_1^2) \\ \Rightarrow t_1 + t_2 &= \frac{-2}{t_1} \\ \Rightarrow t_2 &= -t_1 - \frac{2}{t_1} \end{aligned}$$

88. The conic represented by $x = 2(\cos t + \sin t)$, $y = 5(\cos t - \sin t)$ is

- 1) a circle 2) a parabola
3) an ellipse 4) a hyperbola

Solution : (Correct Answer: 3)

From given equations

$$\begin{aligned} \frac{x}{2} &= \cos t + \sin t \dots (i) \\ \frac{y}{5} &= \cos t - \sin t \dots (ii) \end{aligned}$$

Eliminating t from (i) and (ii), we have $\frac{x^2}{4} + \frac{y^2}{25} = 2$
 $\Rightarrow \frac{x^2}{8} + \frac{y^2}{50} = 1$, which is an ellipse.

89. If the eccentricity of the hyperbola $x^2 - y^2 \operatorname{cosec}^2 \alpha = 25$ is $\sqrt{5}$ times the eccentricity of the ellipse $x^2 \operatorname{cosec}^2 \alpha + y^2 = 5$ then α is equal to :

- 1) $\tan^{-1} \sqrt{2}$ 2) $\sin^{-1} \sqrt{\frac{3}{4}}$
3) $\tan^{-1} \sqrt{\frac{2}{5}}$ 4) $\sin^{-1} \sqrt{\frac{2}{5}}$

Solution : (Correct Answer: 1)

Eccentricity of $\frac{x^2}{25} - \frac{y^2}{25 \sin^2 \alpha} = 1$ is $\sqrt{1 + \sin^2 \alpha}$

Eccentricity of $\frac{x^2}{5 \sin^2 \alpha} + \frac{y^2}{5} = 1$ is $\sqrt{1 - \sin^2 \alpha}$
Given, $\sqrt{1 + \sin^2 \alpha} = \sqrt{5} \sqrt{1 - \sin^2 \alpha}$

$$\Rightarrow \sin^2 \alpha = \frac{2}{3}$$

$$\Rightarrow \alpha = \sin^{-1} \sqrt{\frac{2}{3}} = \tan^{-1} \sqrt{2}$$

90. $\lim_{x \rightarrow 0} \frac{1}{x} \int_0^x (1 + \sin 3t)^{\frac{1}{7}} dt$ is equal to

- 1) 2 2) 1 3) e^2 4) e^3

Solution : (Correct Answer: 4)

$$\lim_{x \rightarrow 0} \frac{\int_0^x (1 + \sin 3t)^{\frac{1}{7}} dt}{x}$$

Differentiating numerator and denominator

$$L = \lim_{x \rightarrow 0} \frac{(1 + \sin 3x)^{\frac{1}{7}}}{1}$$

$$\Rightarrow L = e^{\lim_{x \rightarrow 0} \frac{1}{x} \sin 3x} \Rightarrow L = e^{\lim_{x \rightarrow 0} \frac{3 \sin 3x}{3x}}$$

$$\Rightarrow L = e^{3 \times 1} = e^3$$

91. The logical expression X , in its simplest form for the truth table

a	b	X
1	0	0
1	1	1
0	1	0
0	0	0

is

- 1) $X = a \cdot b$ 2) $X = a + b$
3) $X = a' \cdot b$ 4) $X = a \cdot b'$

Solution : (Correct Answer: 1)

$$X = a \cdot b$$

92. If the normal to the curve $y = f(x)$ at $(3, 4)$ makes an angle $\frac{3\pi}{4}$ with the positive x -axis, then $f'(3)$ is equal to

- 1) -1 2) $\frac{3}{4}$ 3) 1 4) $-\frac{3}{4}$

Solution : (Correct Answer: 3)

Slope of the normal at $(3, 4)$ is the value of

$$-\frac{1}{f'(x)} \text{ at } x = 3 \text{ or } -\frac{1}{f'(3)} = \tan \frac{3\pi}{4} = 1$$

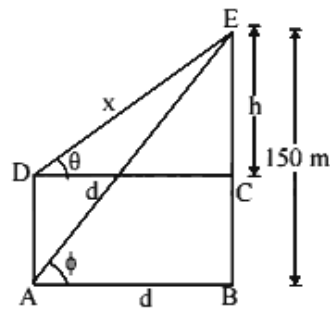
$$\Rightarrow f'(3) = 1$$

93. The angle of depressions of the top and the foot of a chimney as seen from the top of a second chimney, which is 150 m high and standing on the same level as the first are θ and ϕ respectively, then the distance between their tops when $\tan \theta = \frac{4}{3}$ and $\tan \phi = \frac{5}{2}$ is

- 1) $\frac{150}{\sqrt{3}}$ m 2) $100\sqrt{3}$ m
3) 150 m 4) 100 m

Solution : (Correct Answer: 4)

$$\text{Given : } \tan \theta = \frac{4}{3} \text{ and } \tan \phi = \frac{5}{2}$$



$$\text{In } \triangle ABE \tan \phi = \frac{150}{d} \Rightarrow d = 150 \cot \phi = 150 \times \frac{2}{5} = 60 \text{ m}$$

In $\triangle DCE$

$$\tan \theta = \frac{h}{d} \Rightarrow \frac{4}{3} = \frac{h}{d} \Rightarrow h = \frac{4}{3} \times 60 \Rightarrow h = 80 \text{ m}$$

$$\text{Now in } \triangle DCE \quad DE^2 = DC^2 + CE^2 \Rightarrow x^2 = 60^2 + 80^2 = 10000 \Rightarrow x = 100 \text{ m}$$

94. In an equilateral triangle, the inradius, circumradius and one of the exradii are in the ratio

- 1) 2 : 3 : 5 2) 1 : 2 : 3
3) 1 : 3 : 7 4) 3 : 7 : 9

Solution : (Correct Answer: 2)

$$\text{We have, } \Delta = \frac{\sqrt{3}}{4} a^2, s = \frac{3a}{2}$$

$$\text{Inradius } r = \frac{\Delta}{s} = \frac{a}{2\sqrt{3}}$$

$$\text{Circumradius } R = \frac{abc}{4\Delta} = \frac{a^3}{\sqrt{3}a^2} = \frac{a}{\sqrt{3}}$$

$$\text{and exradii } r_1 = \frac{\Delta}{s-a} = \frac{\sqrt{3}/4a^2}{a/2}$$

$$= \frac{\sqrt{3}}{2} a$$

$$\therefore \text{ Required ratio } = r : R : r_1$$

$$= \frac{a}{2\sqrt{3}} : \frac{a}{\sqrt{3}} : \frac{\sqrt{3}}{2} a = 1 : 2 : 3$$

95. In the set $\mathbf{N} \times \mathbf{N}$, the relation R is defined by (a, b) R (c, d) $\Leftrightarrow ad = bc$. Then R is

- 1) partial order relation
2) equivalence relation
3) reflexive and transitive but not symmetric
4) symmetric and transitive but not reflexive

Solution : (Correct Answer: 2)

The relation is defined on $\mathbf{N} \times \mathbf{N}$. since the elements of $\mathbf{N} \times \mathbf{N}$ are ordered pairs hence the relation on $\mathbf{N} \times \mathbf{N}$ relates two ordered pairs. Now,

$$(1) \forall a, b \in \mathbf{N} \quad ab = ba$$

$\therefore (a, b) R (a, b) \forall (a, b) \in \mathbf{N} \times \mathbf{N}$. Thus R is reflexive

(ii) Let

$$(a, b) R (c, d) \Leftrightarrow ad = bc \forall (a, b) \text{ and } (c, d) \in \mathbf{N} \times \mathbf{N} \Rightarrow bc = ad \Rightarrow cb = da \Rightarrow (c, d) R (a, b).$$

Thus R is symmetric

(iii) Again let $(a, b) R (c, d)$ and $(c, d) R (e, f)$

$$\Rightarrow ad = bc \text{ and } cf = de$$

$$\Rightarrow adcf = bcde \Rightarrow af = de \Rightarrow (a, b) R (e, f)$$

Thus R is transitive.

Thus R is equivalence relation.

96. If the normal to the rectangular hyperbola $xy = c^2$ at the point $(ct, \frac{c}{t})$ meets the curve again at $(ct', \frac{c}{t'})$, then

- 1) $t^3 t' = 1$
- 2) $t^3 t' = -1$
- 3) $tt' = 1$
- 4) $tt' = -1$

Solution : (Correct Answer: 2)

The equation of tangent at $(ct, \frac{c}{t})$ is

$$ty = t^3 x - ct^4 + c$$

If it passes through $(ct', \frac{c}{t'})$ then

$$\Rightarrow \frac{tc}{t'} = t^3 ct' - ct^4 + c$$

$$\Rightarrow t = t^3 t'^2 - t^4 t' + t'$$

$$\Rightarrow t \cdot t' = t^3 t' (t' \cdot t) \Rightarrow t^3 t' = -1$$

Note : If we take the co-ordinate axes along the asymptotes of a rectangular hyperbola, then the general equation $x^2 - y^2 = a^2$ becomes $xy = c^2$, where c is a constant.

97. The value of the determinant

$$\begin{vmatrix} \cos \alpha & -\sin \alpha & 1 \\ \sin \alpha & \cos \alpha & 1 \\ \cos(\alpha + \beta) & -\sin(\alpha + \beta) & 1 \end{vmatrix} \text{ is}$$

- 1) independent of α
- 2) independent of β
- 3) independent of α and β
- 4) None of the above

Solution : (Correct Answer: 1)

$$\text{Given, } \begin{vmatrix} \cos \alpha & -\sin \alpha & 1 \\ \sin \alpha & \cos \alpha & 1 \\ \cos(\alpha + \beta) & -\sin(\alpha + \beta) & 1 \end{vmatrix}$$

[Applying $R_3 \rightarrow R_3 - R_1(\cos \beta) + R_2(\sin \beta)$]

$$= \begin{vmatrix} \cos \alpha & -\sin \alpha & 1 \\ \sin \alpha & \cos \alpha & 1 \\ 0 & 0 & 1 + \sin \beta - \cos \beta \end{vmatrix} = (1$$

$+ \sin \beta - \cos \beta)(\cos^2 \alpha + \sin^2 \alpha) = 1 + \sin \beta - \cos \beta$, which is independent of α .

98. The value of $\sin(\cot^{-1}(\cos(\tan^{-1} x)))$ is

- 1) $\sqrt{\frac{x^2+2}{x^2+1}}$
- 2) $\sqrt{\frac{x^2+1}{x^2+2}}$
- 3) $\frac{x}{\sqrt{x^2+2}}$
- 4) $\frac{1}{\sqrt{x^2+2}}$

Solution : (Correct Answer: 2)

We have, $\sin[\cot^{-1}(\cos(\tan^{-1} x))]$

$$= \sin\left[\cot^{-1}\left(\frac{1}{\sqrt{1+\tan^2(\tan^{-1} x)}}\right)\right]$$

$$= \sin\left(\cot^{-1}\frac{1}{\sqrt{1+x^2}}\right)$$

$$= \frac{1}{\sqrt{1+\cot^2\left\{\cot^{-1}\frac{1}{\sqrt{1+x^2}}\right\}}}$$

$$= \frac{1}{\sqrt{1+\frac{1}{1+x^2}}} = \sqrt{\frac{1+x^2}{2+x^2}}$$

99. The domain of the function

$$\sqrt{x^2 - 5x + 6} + \sqrt{2x + 8 - x^2}$$
 is

- 1) $[2, 3]$
- 2) $[-2, 4]$
- 3) $[-2, 2] \cup [3, 4]$
- 4) $[-2, 1] \cup [2, 4]$

Solution : (Correct Answer: 3)

$$f(x) = \sqrt{(x-2)(x-3)} + \sqrt{-(x-4)(x+2)}$$

The first part is real outside $(2, 3)$ and the second is real in $[-2, 4]$ so that the domain is $[-2, 2] \cup [3, 4]$.

100. $f(x) = \begin{cases} x \sin 1/x & , x \neq 0 \\ 0 & , x = 0 \end{cases}$ at $x = 0$ is

- 1) continuous as well as differentiable
- 2) differentiable but not continuous
- 3) continuous but not differentiable
- 4) neither continuous nor differentiable

Solution : (Correct Answer: 3)

For function to be continuous :

$$f(0+h) = f(0-h) = f(0)$$

$$f(0+h) = \lim_{h \rightarrow 0} h \sin 1/h = 0 \times (\text{a finite quantity}) = 0$$

$$f(0-h) = \lim_{h \rightarrow 0} -h \sin(1/-h) = 0 \times (\text{a finite quantity}) = 0$$

$$\text{Also, } \lim_{x \rightarrow 0} x \sin 1/x = 0 \times (\text{a finite quantity}) = 0$$

$\Rightarrow$ function is continuous at $x = 0$

For function to be differentiable :

$$f'(0+h) = f'(0-h)$$

$$f'(0+h) = \frac{f(0+h)-f(0)}{h}$$

$$= \lim_{h \rightarrow 0} \frac{h \sin \frac{1}{h} - 0}{h} = \lim_{h \rightarrow 0} \sin\left(\frac{1}{h}\right)$$

which does not exist.

101. Area enclosed by the curve $\pi [4(x - \sqrt{2})^2 + y^2] = 8$ is

- 1) π
- 2) 2
- 3) 3π
- 4) 4

Solution : (Correct Answer: 4)

The given curve is

$$\pi [4(x - \sqrt{2})^2 + y^2] = 8$$

$$4(x - \sqrt{2})^2 + y^2 = \frac{8}{\pi}$$

$$(x - \sqrt{2})^2 + \left(\frac{y}{2}\right)^2 = \frac{2}{\pi}$$

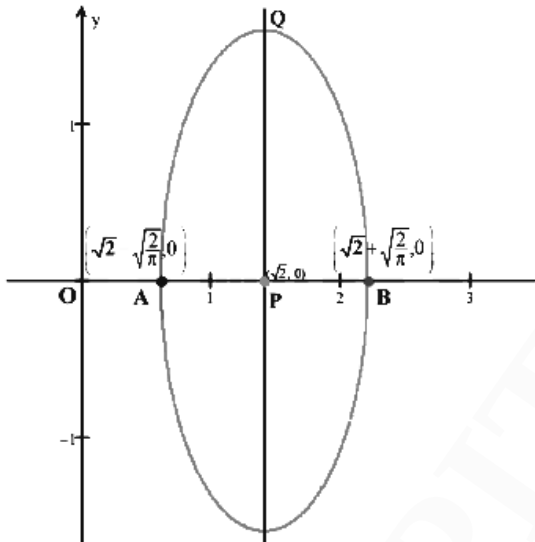
$$\frac{(x - \sqrt{2})^2}{\left(\sqrt{\frac{2}{\pi}}\right)^2} + \frac{y^2}{\left(2\sqrt{\frac{2}{\pi}}\right)^2} = 1 \dots (1)$$

This is the equation of the ellipse having centre $(\sqrt{2}, 0)$

Observe the figure of ellipse (1). The centre

P is $(\sqrt{2}, 0)$. A and B are $(\sqrt{2}, -\sqrt{\frac{2}{\pi}})$ and

$(\sqrt{2} + \sqrt{\frac{2}{\pi}}, 0)$ respectively.



The required area = $4 \times$ area of figure PQB

$$= 4 \times \int_{\sqrt{2}}^{\sqrt{2} + \sqrt{\frac{2}{\pi}}} y dx$$

$$= 4 \times \int_{\sqrt{2}}^{\sqrt{2} + \sqrt{\frac{2}{\pi}}} \sqrt{\frac{8}{\pi} - 4(x - \sqrt{2})^2} dx$$

$$= 4 \times 2 \int_{\sqrt{2}}^{\sqrt{2} + \sqrt{\frac{2}{\pi}}} \sqrt{\left(\sqrt{\frac{2}{\pi}}\right)^2 - (x - \sqrt{2})^2} dx$$

$$= 8 \left[\frac{x - \sqrt{2}}{2} \sqrt{\frac{2}{\pi} - (x - \sqrt{2})^2} + \frac{(2/\pi)}{2} \sin^{-1} \left(\frac{x - \sqrt{2}}{\sqrt{2/\pi}} \right) \right]_{\sqrt{2}}^{\sqrt{2} + \sqrt{\frac{2}{\pi}}}$$

$$\left[\because \int \sqrt{a^2 - x^2} dx = \frac{1}{2} x \sqrt{a^2 - x^2} + \frac{a^2}{2} \sin^{-1} \frac{x}{a} + C \right]$$

$$= 8 \left[\left\{ \frac{\sqrt{2} + \sqrt{\frac{2}{\pi}} - \sqrt{2}}{2} \sqrt{\frac{2}{\pi} - \left(\sqrt{2} + \sqrt{\frac{2}{\pi}} - \sqrt{2} \right)^2} \right. \right.$$

$$\left. + \frac{1}{\pi} \sin^{-1} \left(\frac{\sqrt{2} + \sqrt{\frac{2}{\pi}} - \sqrt{2}}{\sqrt{\frac{2}{\pi}}} \right) \right\} - \left\{ \frac{\sqrt{2} - \sqrt{2}}{2} \sqrt{\frac{2}{\pi} - (\sqrt{2} - \sqrt{2})^2} + \frac{1}{\pi} \sin^{-1} \left(\frac{\sqrt{2} - \sqrt{2}}{\sqrt{2/\pi}} \right) \right\} \right]$$

$$= 8 \left[\frac{1}{\sqrt{2\pi}} (0) + \frac{1}{\pi} \sin^{-1}(1) - 0 - 0 \right]$$

$$= 8 \left(\frac{1}{\pi} \times \frac{\pi}{2} \right) = 4 \text{ square units.}$$

102. If the tangent to the function $y = f(x)$ at $(3, 4)$ makes an angle of $\frac{3\pi}{4}$ with the positive direction of x -axis in anticlockwise direction then $f'(3)$ is

- 1) -1 2) 1 3) $\frac{1}{\sqrt{3}}$ 4) $\sqrt{3}$

Solution : (Correct Answer: 1)

$$f'(3) = \tan \frac{3\pi}{4} = -\tan \frac{\pi}{4} = -1$$

103. The value of the integral

$$\int_{-1/2}^{1/2} \left[\left(\frac{x+1}{x-1} \right)^2 + \left(\frac{x-1}{x+1} \right)^2 - 2 \right]^{1/2} dx \text{ is}$$

- 1) $\log \left(\frac{4}{3} \right)$ 2) $4 \log \left(\frac{3}{4} \right)$
3) $4 \log \left(\frac{4}{3} \right)$ 4) $\log \left(\frac{3}{4} \right)$

Solution : (Correct Answer: 3)

$$\int_{-1/2}^{1/2} \left[\left(\frac{x+1}{x-1} \right)^2 + \left(\frac{x-1}{x+1} \right)^2 - 2 \right]^{1/2} dx$$

$$= \int_{-1/2}^{1/2} \left[\left(\frac{x+1}{x-1} - \frac{x-1}{x+1} \right)^2 \right]^{1/2} dx$$

$$= \int_{-1/2}^{1/2} \left| \frac{4x}{x^2-1} \right| dx$$

$$= \int_{-1/2}^0 \left| \frac{4x}{1-x^2} \right| dx + \int_0^{1/2} \left| \frac{4x}{1-x^2} \right| dx$$

$$= -4 \int_{-1/2}^0 \frac{x}{1-x^2} dx + 4 \int_0^{1/2} \frac{x}{1-x^2} dx$$

$$= 2 \{ \log(1-x^2) \}_{-1/2}^0 - 2 \{ \log(1-x^2) \}_0^{1/2}$$

$$= -2 \log \left(1 - \frac{1}{4} \right) - 2 \log \left(1 - \frac{1}{4} \right)$$

$$= -4 \log \frac{3}{4} = 4 \log \frac{4}{3}$$

104. Evaluate $\int \frac{x^2+4}{x^4+16} dx$.

- 1) $\frac{1}{2\sqrt{2}} \tan^{-1} \left(\frac{x^2-4}{2x\sqrt{2}} \right) + c$
2) $\frac{1}{2\sqrt{2}} \tan^{-1} \left(\frac{x^2-4}{2\sqrt{2}} \right) + c$
3) $\frac{1}{2\sqrt{2}} \tan^{-1} \left(\frac{x^2-4}{x\sqrt{2}} \right) + c$
4) None of the above

Solution : (Correct Answer: 1)

$$\text{Let } I = \int \frac{x^2+4}{x^4+16} dx = \int \frac{1 + \frac{4}{x^2}}{x^2 + \frac{16}{x^2}} dx = \int \frac{1 + \frac{4}{x^2}}{\left(x - \frac{4}{x}\right)^2 + 8} dx$$

$$\text{Putting } x - \frac{4}{x} = t$$

$$\text{So that } \left(1 + \frac{4}{x^2} \right) dx = dt$$

$$\therefore I = \int \frac{dt}{t^2 + (2\sqrt{2})^2}$$

$$= \frac{1}{2\sqrt{2}} \tan^{-1} \left(\frac{t}{2\sqrt{2}} \right) + C$$

$$\Rightarrow I = \frac{1}{2\sqrt{2}} \tan^{-1} \left(\frac{x - \frac{4}{x}}{2\sqrt{2}} \right) + C$$

$$= \frac{1}{2\sqrt{2}} \tan^{-1} \left(\frac{x^2-4}{2x\sqrt{2}} \right) + C$$

105. The chances of defective screws in three boxes A, B, C are $\frac{1}{5}$, $\frac{1}{6}$, $\frac{1}{7}$ respectively. A box is selected at random and a screw drawn from it at random is found to be defective. Then, the probability that it came from box A, is

1) $\frac{16}{29}$ 2) $\frac{1}{15}$ 3) $\frac{27}{59}$ 4) $\frac{42}{107}$

Solution : (Correct Answer: 4)

Let E_1 , E_2 and E_3 denote the events of selecting boxes A, B, C respectively and A be the event that a screw selected at random is defective. Then,

$$P(E_1) = \frac{1}{3}, P(E_2) = \frac{1}{3}, P(E_3) = \frac{1}{3}$$

$$P\left(\frac{A}{E_1}\right) = \frac{1}{5}, P\left(\frac{A}{E_2}\right) = \frac{1}{6}, P\left(\frac{A}{E_3}\right) = \frac{1}{7}$$

By Baye's rule, the required probability

$$P\left(\frac{E_1}{A}\right) = \frac{P(E_1)P\left(\frac{A}{E_1}\right)}{P(E_1)P\left(\frac{A}{E_1}\right) + P(E_2)P\left(\frac{A}{E_2}\right) + P(E_3)P\left(\frac{A}{E_3}\right)}$$

$$\Rightarrow P\left(\frac{E_1}{A}\right) = \frac{\frac{1}{3} \cdot \frac{1}{5}}{\frac{1}{3} \cdot \frac{1}{5} + \frac{1}{3} \cdot \frac{1}{6} + \frac{1}{3} \cdot \frac{1}{7}}$$

$$= \frac{\frac{1}{5}}{\frac{1}{5} + \frac{1}{6} + \frac{1}{7}} = \frac{42}{107}$$

106. The solution of the differential equation $\log x \frac{dy}{dx} + \frac{y}{x} = \sin 2x$ is

- 1) $y \log |x| = C - \frac{1}{2} \cos x$
- 2) $y \log |x| = C + \frac{1}{2} \cos 2x$
- 3) $y \log |x| = C - \frac{1}{2} \cos 2x$
- 4) $xy \log |x| = C - \frac{1}{2} \cos 2x$

Solution : (Correct Answer: 3)

$$\frac{dy}{dx} + \frac{y}{x \log x} = \frac{\sin 2x}{\log x}$$

$$\text{I.F.} = e^{\int \frac{dx}{x \log x}}$$

$$\therefore \text{I.F.} = e^{\int \frac{1}{t} dt} = e^{\log t} = t = \log |x|$$

solution is given by

$$y(\text{I.F.}) = \int Q(\text{I.F.})dx + C$$

$$y \log |x| = \int \frac{\sin 2x}{\log |x|} (\log |x|) dx + C$$

$$= -\frac{\cos 2x}{2} + C$$

107. The length of the parabola $y^2 = 12x$ cut off by the latus-rectum is

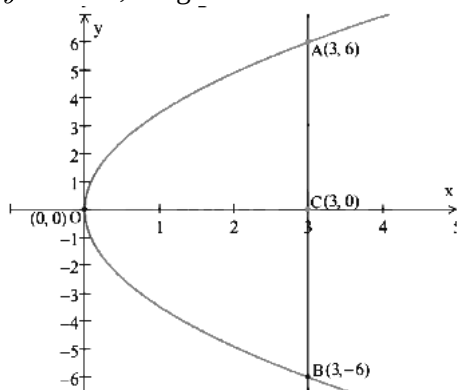
- 1) $6(\sqrt{2} + \log(1 + \sqrt{2}))$
- 2) $3(\sqrt{2} + \log(1 + \sqrt{2}))$
- 3) $6(\sqrt{2} - \log(1 + \sqrt{2}))$
- 4) $3(\sqrt{2} - \log(1 + \sqrt{2}))$

Solution : (Correct Answer: 1)

On comparing the equation of the parabola

$y^2 = 12x$ with the standard equation,

$y^2 = 4ax$, we get $4a = 12$ or $a = 3$



Hence, the focus, point C will be at (3, 0) and the extremities of the latus-rectum AB will be at (a, 2a) and (a, -2a). So the coordinates of A and B are (3, 6) and (3, -6) respectively. Now we need to find the length (curve AOB) of the parabola. As it is not a straight line so we cannot directly find the length of

this curve as we cannot directly apply Pythagorean theorem. Let us consider a small length ds on the parabola. Using Pythagorean theorem for this length,

$$ds = \sqrt{(dx)^2 + (dy)^2} = \sqrt{\left(\frac{dx}{dy}\right)^2 + 1} dy$$

$$\Rightarrow s = \int_{-6}^6 \left[\sqrt{\left(\frac{dx}{dy}\right)^2 + 1} \right] dy \dots (1)$$

$$\text{From } y^2 = 12x \Rightarrow x = \frac{y^2}{12}$$

$$\frac{dx}{dy} = \frac{2y}{12} = \frac{y}{6} \quad \text{Putting in (1),}$$

$$s = \int_{-6}^6 \left[\sqrt{\left(\frac{y}{6}\right)^2 + 1} \right] dy = 2 \int_0^6 \sqrt{\frac{y^2 + 36}{36}} dy$$

$$= \frac{2}{6} \int_0^6 \sqrt{y^2 + 6^2} dy$$

$$\text{Using } \int \sqrt{x^2 + a^2} = \frac{x}{2} \sqrt{a^2 + x^2}$$

$$+ \frac{a^2}{2} \log(x + \sqrt{a^2 + x^2}) + C$$

$$\text{We get } s = \frac{1}{3} \left[\frac{y}{2} \sqrt{6^2 + y^2} + \frac{6^2}{2} \right]$$

$$\log(y + \sqrt{6^2 + y^2}) + C \Big|_0^6$$

$$= \frac{1}{3} \left[\frac{6}{2} \sqrt{6^2 + 6^2} + 18 \log(6 + \sqrt{6^2 + 6^2}) \right]$$

$$+ C - 0 - 18 \log(0 + \sqrt{6^2 + 0}) - C$$

$$= \frac{1}{3} [3.6\sqrt{2} + 18 \log(6 + 6\sqrt{2}) - 18 \log 6]$$

$$= 6\sqrt{2} + 6 \log \frac{6(1+\sqrt{2})}{6}$$

$$= 6[(\sqrt{2} + \log(1 + \sqrt{2}))]$$

108. The line which passes through the origin and intersect the two lines $\frac{x-1}{2} = \frac{y+3}{4} = \frac{z-5}{3}$, $\frac{x-4}{2} = \frac{y+3}{3} = \frac{z-14}{4}$, is

$$1) \frac{x}{1} = \frac{y}{-3} = \frac{z}{5}$$

$$2) \frac{x}{-1} = \frac{y}{3} = \frac{z}{5}$$

$$3) \frac{x}{1} = \frac{y}{3} = \frac{z}{-5}$$

$$4) \frac{x}{1} = \frac{y}{4} = \frac{z}{-5}$$

Solution : (Correct Answer: 1)

Let the line be $\frac{x}{a} = \frac{y}{b} = \frac{z}{c} \dots (i)$

If line (i) intersects with the line $\frac{x-1}{2} = \frac{y+3}{4} = \frac{z-5}{3}$, then

$$\begin{vmatrix} a & b & c \\ 2 & 4 & 3 \\ 4 & -3 & 14 \end{vmatrix} = 0$$

$$\Rightarrow 9a - 7b - 10c = 0 \dots (ii)$$

from (i) and (ii), we have

$$\frac{a}{1} = \frac{b}{-3} = \frac{c}{5}$$

$$\therefore \text{The line is } \frac{x}{1} = \frac{y}{-3} = \frac{z}{5}$$

109. Convert the hexadecimal numeral ABCD into binary numeral

$$1) -1010101111001100$$

$$2) (1001000011111111)_2$$

$$3) (1111110000010001)_2$$

$$4) (1000100100111100)_2$$

Solution : (Correct Answer: 1)

Replacing each hexadecimal digit by the corresponding 4-digit binary numeral, we have

$$(ABCD)_{16} = (1010101111001101)_2$$

110. If X is a poisson variate such that $P(X = 1) = P(X = 2)$, then $P(X = 4)$ is equal to

- 1) $\frac{1}{2e^2}$ 2) $\frac{1}{3e^2}$
 3) $\frac{2}{3e^2}$ 4) $\frac{1}{e^2}$

Solution : (Correct Answer: 3)

Given : $P(X = 1) = P(X = 2)$

$$\frac{e^{-\lambda} \lambda}{1!} = \frac{e^{-\lambda} \lambda^2}{2!}$$

$$\Rightarrow \lambda = 2$$

$$\therefore P(X = 4) = \frac{e^{-2} 2^4}{4!} = \frac{2}{3e^2}$$

English - Section A (MCQ.)

111. Earth has not anything to show more fair. Dull would he be of soul who could pass doth like a garment wear. The beauty of the morning; silent, bare, Ships, towers, domes, theatres and temples lie Open unto the fields and to the sky; All bright and glittering in the smokeless air. Never did Sun more beautifully steep. In his first splendour valley, rock or hill; Ne'er saw I, never felt a calm so deep! The river glideth at his own sweet will: De mighty heart is lying still.

The narrator call a person dull if he/she

- 1) does not talk to him openly 2) does not know about romantic sites 3) passes by without experiencing the beauty of the area. 4) passes over the site and shouts loudly

Solution : (Correct Answer: 3)

The answer is Option C. Here's why:

The line in the poem states, "Dull would he be of soul who could pass by / A sight so touching in its majesty."

This means that the narrator believes any person who can walk past such a beautiful sight without truly experiencing or appreciating it must have a dull soul. Thus, the narrator is referring to someone who passes by without experiencing the beauty of the area.

112. Pick out the correct synonym of the word 'Solitude'.

- 1) Musical composition 2) Aloneness 3) Time statement 4) Single-mindedness

Solution : (Correct Answer: 2)

The correct synonym of "solitude" is:

Option B: Aloneness

Explanation:

"Solitude" means being alone or enjoying one's own company.

"Aloneness" directly conveys this meaning, making it the best match.

The other options do not capture the sense of being alone.

113. Select the antonym of the word 'Gregarious'.

- 1) Antisocial 2) Horrendous 3) Glorious 4) Similar

Solution : (Correct Answer: 1)

The word "gregarious" means sociable or fond of company. Its antonym would be a word that means unsociable or averse to social interaction. Here's how the options break down:

Option A: Antisocial – Means not wanting to be social or friendly.

Option B: Horrendous – Means extremely unpleasant, but it doesn't relate to sociability.

Option C: Glorious – Means splendid or wonderful, which isn't related.

Option D: Similar – Means alike, which doesn't apply here.

Since "antisocial" directly opposes "gregarious," the correct answer is:

Option A: Antisocial.

114. Earth has not anything to show more fair. Dull would he be of soul who could pass by A sight so touching in its majesty. This city now doth like a garment wear. The beauty of the morning; silent, bare, Ships, towers, domes, theatres and temples lie Open unto the field and to the sky; All bright and glittering in the smokeless air. Never did Sun more beautifully steep. In his first splendour valley, rock or hill; Ne'er saw I, never felt a calm so deep! The river glideth at his own sweet will: Dear God! The very houses seem asleep; And all that mighty heart is lying still.

The city of London appear as if it is _____

- 1) covered with thick layer of smoke 2) clad in attractive colourful clothing 3) desolate and lonely 4) sleeping beauty

Solution : (Correct Answer: 4)

The correct answer is Option D – sleeping beauty. Here's why:

The poem describes London early in the morning, emphasizing its calm and peaceful appearance. Phrases like "the very houses seem asleep" and "all that mighty heart is lying still" suggest that the city appears calm and undisturbed, much like someone who is sleeping.

This imagery is what leads us to the idea of London being compared to a "sleeping beauty."

Aptitude - Section A (MCQ.)

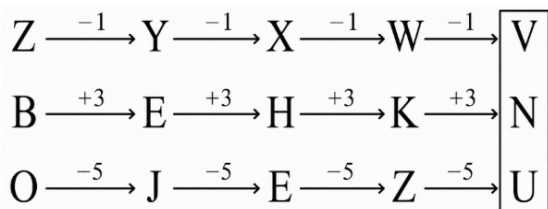
115. From the given options, choose the correct one that will replace the question mark (?) in the following series.

ZBO, YEJ, XHE, WKZ, ?

- 1) VNU 2) INU 3) ZXP 4) VUN

Solution : (Correct Answer: 1)

Pattern of the series is as follow.



$\therefore ? = \text{VNU}$

116. The questions given below consist of a question followed by two statements labelled as I and II. We have to decide whether these statements given enough information required to answer the question or not.

Question in which year was Ritika born?

Statement

I. Ritika at present is 25 years younger to her mother.

II. Ritika's sister who was born in 1964, is 35 years younger to her mother.

Give answer

- 1) I alone is sufficient while II alone is not sufficient. 2) II alone is sufficient while I alone is not sufficient. 3) Either I or II is sufficient. 4) Both I and II are sufficient.

Solution : (Correct Answer: 4)

Let's analyze the information given in each statement to see if we have enough to determine Ritika's birth year.

Statement I: "Ritika at present is 25 years younger to her mother."

This tells us that the age difference between Ritika and her mother is 25 years.

In equation form, if we let the mother's birth year be M and Ritika's birth year be R , then:

$$R = M + 25$$

However, without knowing the actual birth year of the mother (or any specific year), we cannot determine R uniquely.

Thus, Statement I by itself is not sufficient.

Statement II: "Ritika's sister who was born in 1964, is 35 years younger to her mother."

This statement allows us to determine the mother's birth year.

If the sister was born in 1964 and is 35 years younger than her mother, then their birth years differ by 35 years:

$$M = 1964 - 35 = 1929$$

Now we know the mother was born in 1929.

However, Statement II does not tell us anything about Ritika's age or her relation to the mother.

Without knowing the 25-year difference between Ritika and her mother (as given in Statement I), we cannot find Ritika's birth year based solely on Statement II.

Therefore, Statement II by itself is not sufficient.

Combine Statements I and II:

From Statement II, we already calculated the mother's birth year as 1929.

Now, using Statement I, where Ritika is 25 years younger than her mother, we find:

$$R = 1929 + 25 = 1954$$

With both pieces of information, we can uniquely determine that Ritika was born in 1954.

Conclusion: Neither statement alone is sufficient, but when combined they provide enough information to answer the question.

The correct answer is: Option D – Both I and II are sufficient.

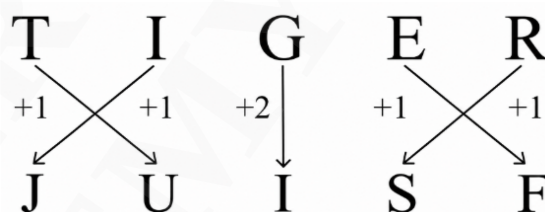
117. In a code language, 'TIGER' is written as 'JUISF'.

How will 'EQUAL' be written as in that language?

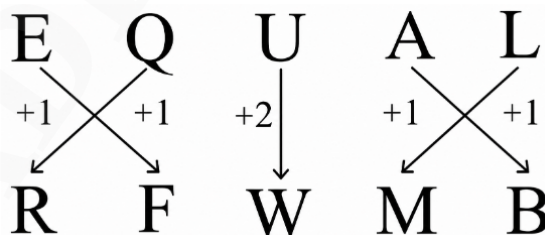
- 1) RFXMB 2) RFWMB 3) RXMBF 4) RFWBE

Solution : (Correct Answer: 2)

As,



Similarly,

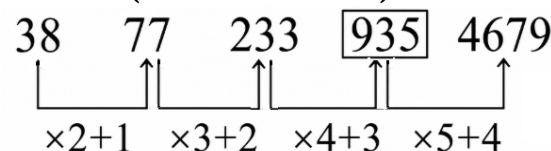


118. Select the number from among the given options that can replace the question mark (?) in the following series.

38, 77, 233, ?, 4679

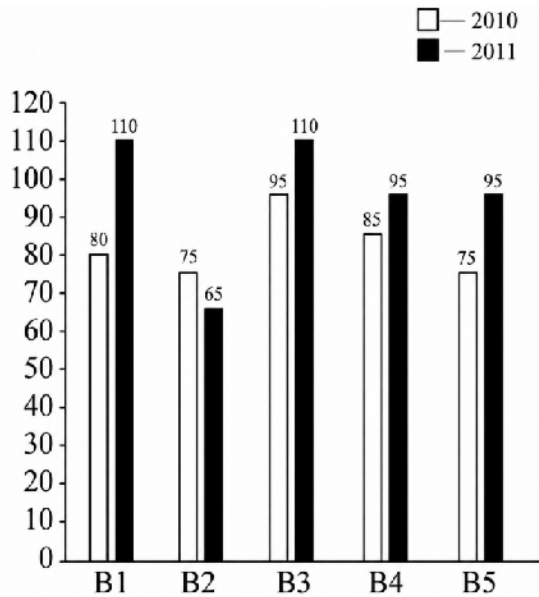
- 1) 641 2) 842 3) 699 4) 935

Solution : (Correct Answer: 4)



$\therefore ? = 935$

119. Following bar graph shows the sales of books (in thousand number) from Five branches of a publishing company during two consecutive years 2010 and 2011.



What is the ratio of total sale of branch B_2 for both year to the total sale of branch B_4 for both year?

- 1) 2 : 3 2) 3 : 5 3) 7 : 9 4) 4 : 5

Solution : (Correct Answer: 3)

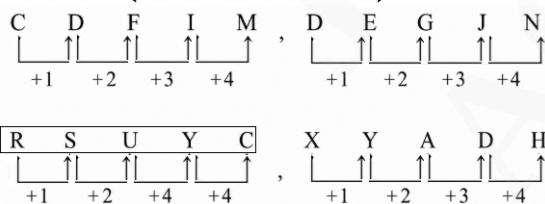
$$\frac{\text{Total sale of Branch } B_2}{\text{Total sale of Branch } B_4} = \frac{75+65}{85+95}$$

$$\text{Required ratio} = \frac{140}{180} = \frac{7}{9}$$

120. Four letter-clusters have been given, out of which three are alike in same manner and one is different. Select the letter cluster that is different.

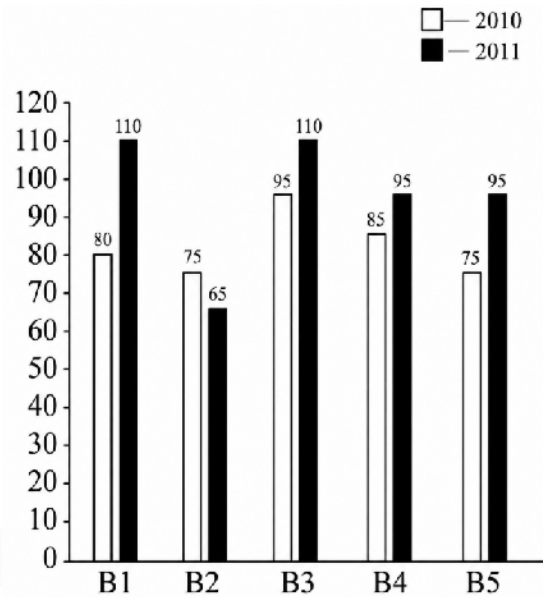
- 1) CDFIM 2) DEGJN 3) RSUYC 4) XYADH

Solution : (Correct Answer: 3)



∴ RSUYC is different.

121. Following bar graph shows the sales of books (in thousand number) from Five branches of a publishing company during two consecutive years 2010 and 2011.



What is the average sales of all the branches (in thousand numbers) for the year of 2010?

- 1) 85 2) 82 3) 90 4) 93

Solution : (Correct Answer: 2)

According to the question, Average sales of all the five branches (in thousand number) for the year 2010

$$= \frac{80+75+95+85+75}{5}$$

$$= \frac{410}{5} = 82$$